§6.1 (We'll go back to Ch 5)

3/24/2011

Section 10

Last time:

θ radian measure of an angle = arclength cut/swept out on the unit circle by θ.

Formulas: (not provided on final; will be provided on test)

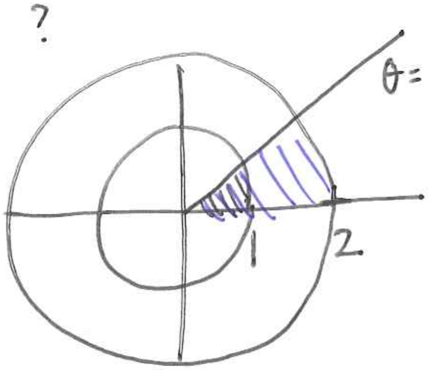
$$s = r\theta$$

θ measured in radians.

$$A_\theta = \text{sector area} = \tfrac{1}{2}\theta r^2.$$

Why A_θ?

$\theta = \pi/4 = \frac{1}{8}$ of rev.



$A_{circle} = \pi r^2$

$A_{rad\ 1} = \pi$ $\qquad A_{\pi/4 \text{ on rad } 1} = \frac{\pi}{8} = \frac{1}{2}\frac{\pi}{4}$

$A_{rad\ 2} = 4\pi$ $\qquad A_{\pi/4 \text{ on rad } 2} = \frac{\pi}{2} = \frac{1}{2}\frac{\pi}{4}4$

$$\Rightarrow A_\theta = \tfrac{1}{2}\theta r^2$$

$$\frac{1}{8}\text{ rev} = \frac{\frac{\pi}{4}}{2\pi}$$

Idea:

$$\#\text{ of full revs in }\theta = \frac{\theta}{2\pi} = \frac{S_\theta}{2\pi r} = \frac{A_\theta}{\pi r^2}$$

Example! Find the area and arclength swept out by on a circle of radius 10.

ⓐ $\theta = 120°$

1.) convert to radian

$$120° \left(\frac{\pi}{180°}\right) = \frac{2}{3}\pi$$

2.) Evaluate:

$$A_\theta = \frac{1}{2}\theta r^2$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3}\pi\right)(10)^2$$

$$A_\theta = \frac{100}{3}\pi$$

$$S_\theta = r\theta$$

$$= 10\left(\frac{2}{3}\pi\right)$$

$$S_\theta = \frac{20}{3}\pi$$

ⓑ $\theta = R$ radians

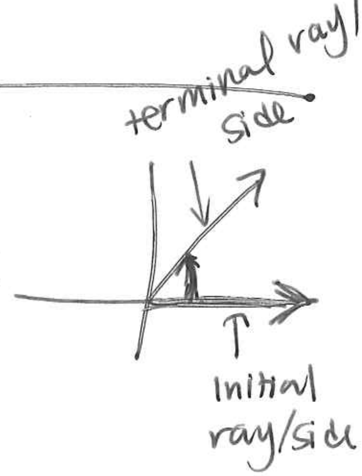
1.) already in rad.

2.) Evaluate

$$A_\theta = \frac{1}{2}R(10)^2 = 50R$$

$$S_\theta = R(10) = 10R$$

Terminology / definitions:



An angle in standard position has an initial ray of the positive horiz axis and its vertex is at the origin.

If an angle is negative, rotate clockwise.
" " positive, rotate counterclockwise.

Two angles in standard position are coterminal if their terminal sides are the same.

Ex: $0°$ and $360°$ $(720°, 1080°, \ldots)$

are all coterminal. NOT same angle.

they just have same terminal side.

Def:

linear velocity (speed along one direction)

usual definition of velocity. $\frac{\text{distance}}{\text{time}}$.

$$v = \frac{s}{t} \quad \left(\frac{\text{distance}}{\text{time}}\right)$$

angular velocity (measures how quickly revolving).

$$\omega = \frac{\theta}{t} \quad \left(\frac{\text{angle}}{\text{time}}\right)$$

Example 7 in ~~text cousin~~:

Suppose you are riding 27 inch diam wheels at a rate of 15 miles per hour.

Find the angular speed of the wheel.

(WA units are also annoying!)

$\frac{15 \text{ miles}}{1 \text{ hour}} \Rightarrow$ in 1 hour,

15 miles =

$$15 \cancel{mi} \left(\frac{5280 \cancel{ft}}{1 \cancel{mi}}\right)\left(\frac{12 \text{ in}}{1 \cancel{ft}}\right)$$

$= 950400$ inches

circumference $= \cancel{2}\pi\left(\frac{27}{\cancel{2}}\right)$

27π in

# of full revs to travel 15 miles $= \frac{950{,}400 \text{ in}}{27\pi} = 11{,}204.5$ revs

angle in 1 hour (measured in rad) $= 11{,}204.5 \text{ revs} \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)$

$= 70{,}400$ rad in 1 hour.

$\omega = 70{,}400 \text{ rad/hr}$

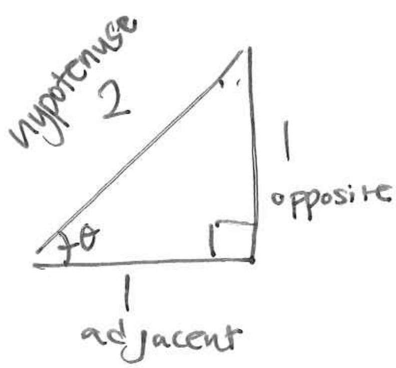
ang vel in rad per sec.

$70{,}400 \frac{\text{rad}}{\text{hr}} \cdot \left(\frac{1 \text{ hr}}{60 \text{ min}}\right)\left(\frac{\text{min}}{60 \text{ sec}}\right)$

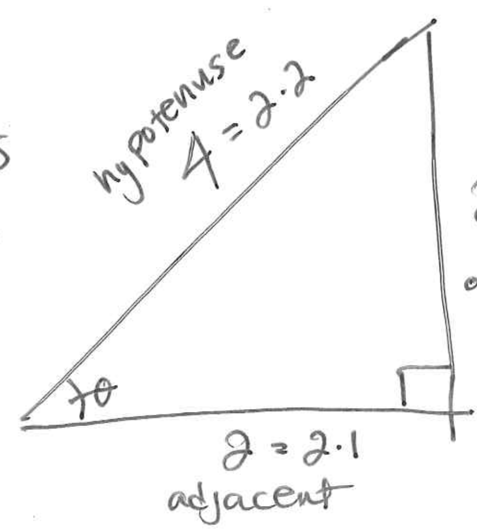
$\boxed{19.56 \text{ rad/sec}}$

$v = r\omega$ rel b/t angular and linear velocity.

$\boxed{\S 6.2}$ Right triangles:



Two times bigger



Similar triangles

$\frac{\text{opp}}{\text{adj}} = \frac{1}{1} = \frac{\cancel{2}\cdot 1}{\cancel{2}\cdot 1} = \tan(\theta) =$ Draw any right triangle w/ an angle θ in it, take ratio $\frac{\text{opp}}{\text{adj.}}$

In general: Given an angle θ $\quad \underbrace{0 < \theta < 90^\circ = \frac{\pi}{2}}_{\text{"acute"}}$

define

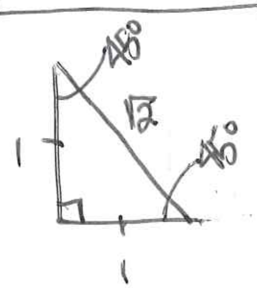
$\sin\theta = \frac{opp}{hyp}$ "SOH $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

$\cos\theta = \frac{adj}{hyp}$ CAH $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

$\tan\theta = \frac{opp}{adj}$ TOA" $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta}$

Know for test!

Special Triangles:



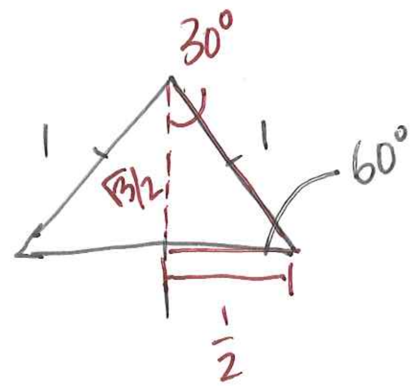
$1^2 + 1^2 = h^2$

$2 = h^2$

$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 45^\circ = 1$



$(\frac{1}{2})^2 + y^2 = 1^2$

$y^2 = \frac{3}{4}$

$y = \frac{\sqrt{3}}{2}$

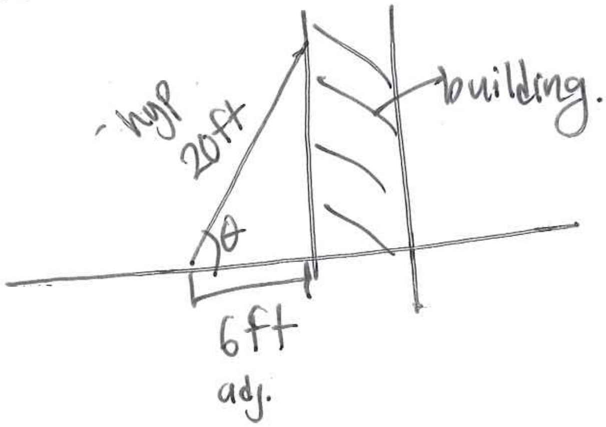
$\sin 60^\circ = \sqrt{3}/2$ | $\sin 30^\circ = \frac{1}{2}$

$\cos 60^\circ = \frac{1}{2}$ | $\cos 30^\circ = \sqrt{3}/2$

$\tan 60^\circ = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$ | $\tan 30^\circ = \frac{\frac{1}{2}}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$.

Easy to work w/ hyp 1 triangles.

Applications: #50 in text:



Find θ:

$$\cos(\theta) = \frac{6}{20}$$

On calc, use $\cos^{-1}$ (inv cosine) to find θ.

$\theta \approx 72.5^\circ$

≈ 1.27 rad.

In class exercise:

#7, 11 in text.

#55 in text.