§2.4 Transformations.

Th 2/3
Section 10

Examples:

Given $f(x)$ as a table below

x	0	3	6	9
$f(x) = y$	4	8	16	24

Find tables for:

(a) $y = f(x) + 1$

x	0	3	6	9
y	5	9	17	25

← add 1 to old y-values.

(b) $y = f(x-2)$

$f(5-2) = f(3)$ = known value in table.
$f(8-2) = f(6)$ = "

x	2	5	8	11
$f(x-2)$	4	8 / 0	16	24

← increased add by 2

(c) $y = 2f(x)$

x	0	3	6	9
$2f(x)$	8 (2·4)	16 (2·8)	32 (2·16)	48 (2·24)

— nothing done to x.

← double the y's

(d) $y = f(3x)$

x	0	1	2	3
y	4	8	16	24

← divide x values by 3.

← nothing changes

In general, given either addition (sub) or mult (division) by a constant on the inside or outside of a function, the corresponding inputs or outputs change in a predictable way.

Arithmetic on the outside yields the same arithmetic on y-values.

Arithmetic on the inside yields the opposite arithmetic on x-values.

Example: Describe what each formula will do to the base $y=f(x)$.

(a) $f(x-5)$ changes the inputs (y vals stay same) by adding 5 to inputs.

(b) $3f(x)+5$ changes the outputs (x vals stays the same) by first triple y's then add 5.

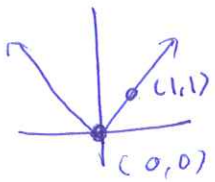
(c) $3(f(x)+5)$ changes the outputs by adding 5 then tripling.

Definition: Given a function $f(x)$, a transformation of $f(x)$ is a function of the form

$$y = Af(Bx+C)+D \quad , \; A, B, C, D \text{ are constants.}$$

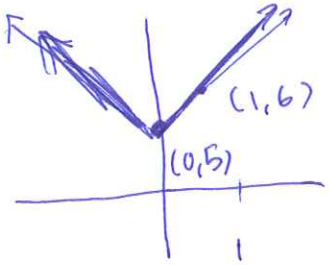
Connection to graphs:

$y = |x|$ base graph.



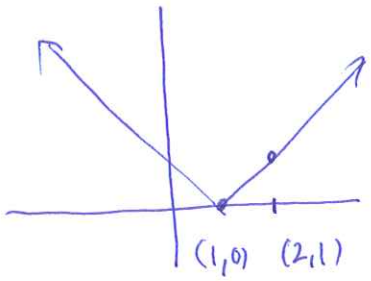
Sketch

(a) $y = |x| + 5$



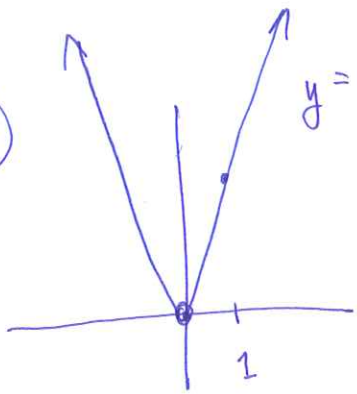
vert shift up.

(b) $y = |x-1|$ ← add 1 to inputs



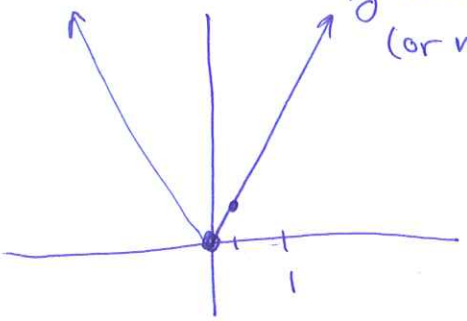
Horiz shift to R by one unit.

(c) $y = 3|x|$ ← changes outputs



Vertical stretch/expansion by a factor of 3.

(d) $y = |3x|$ ← changes inputs by dividing 3 ~~or mult~~ (or mult by $\frac{1}{3}$)



horiz shrink/compression by a factor of $\frac{1}{3}$.

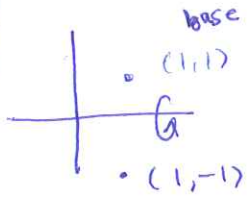
Summary of possible basic transformations

1.) $f(x)+C$: changes outputs by adding C.
Vertical shift (up if $C>0$
down if $C<0$).

2.) $Af(x)$: changes outputs by mult by A
Vertical stretch/exp : $A>1$
Vertical shrink/comp : $0<A<1$

3.) $-f(x)$: changes sign of outputs
$\Rightarrow$ reflection over x-axis.

outside transformations



=

4.) $f(x+C)$: changes inputs by subtracting C
horizontal shift.

5.) $f(Bx)$: changes inputs by dividing by B -
horiz compression/expansion.

6.) $f(-x)$: changes sign on the input
$\Rightarrow$ reflection over y-axis.

inside ~~transside~~ transformations