2/24/2011
Section 10

$\boxed{\S 3.1 - 3.3}$ Polynomials.

$$y = \boxed{a_n x^n} + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

lead term

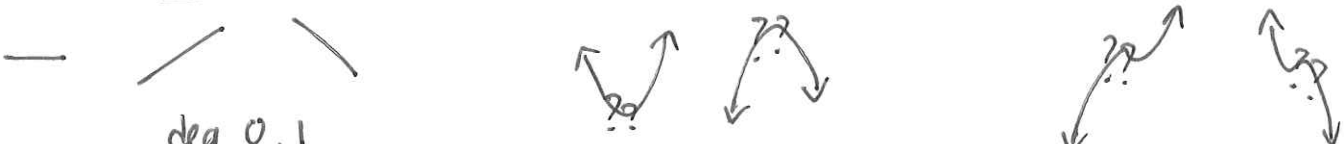
Lead Coeff

yint (constant term)

Lead term prop:

$f(x) = a_n x^n + \cdots + a_1 x + a_0$ has same end behavior as its lead term $a_n x^n$.

line | u-shape (even deg) | s-shape (odd deg)



deg 0, 1 | $a_n > 0$ | $a_n < 0$ | $a_n > 0$ | $a_n < 0$

Graph properties: poly graphs must:

- domain is all real #s.
- graph is continuous (can draw graph w/o lifting pencils)
  no holes, no breaks, no jumps.
- graph is smooth.
  no corners, pinches/cusps.

Facts about degree: If poly is of degree n

$n \geq$ # of zeros/x intercepts.

$n-1 \geq$ # of Turning points

Zeros and their multiplicity:

Def: A zero $x=c$ of a polynomial has multiplicity m if $(x-c)^m$ is the largest power of $x-c$ that divides the polynomial.

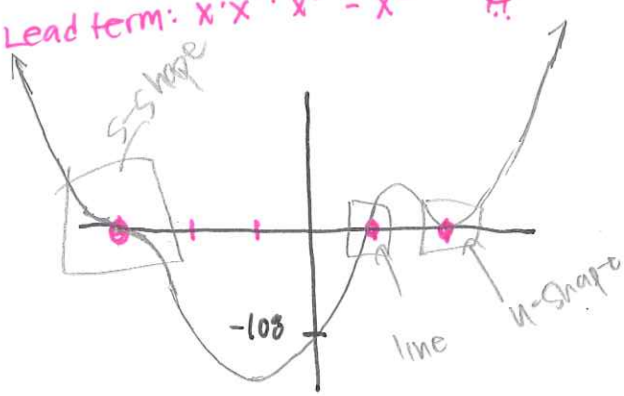
Shortcut: Factor the poly completely. The exponent on $(x-c)$ is the multiplicity.

Example 1 Given the poly; Find its zeros, their multiplicities and sketch.

ⓐ $y = (x-1)(x-2)^2(x+3)^3$ → Lead term: $x \cdot x^2 \cdot x^3 = x^6$

zeros: $x = 1, 2, -3$

$x=1$	$x=2$	$x=-3$
mult 1	mult 2	mult 3.
line @ xint	u-shape @ x-int	s-shape @ x-int
$(1,0)$	$(2,0)$	$(-3,0)$



yint: $(-1)(-2)^2(3)^3 = -4(27)$
$= -108$

ⓑ $y = 2x^5 - x^4 - x^3 = x^3(2x^2 - x - 1) = x^3(x-1)(2x+1)$

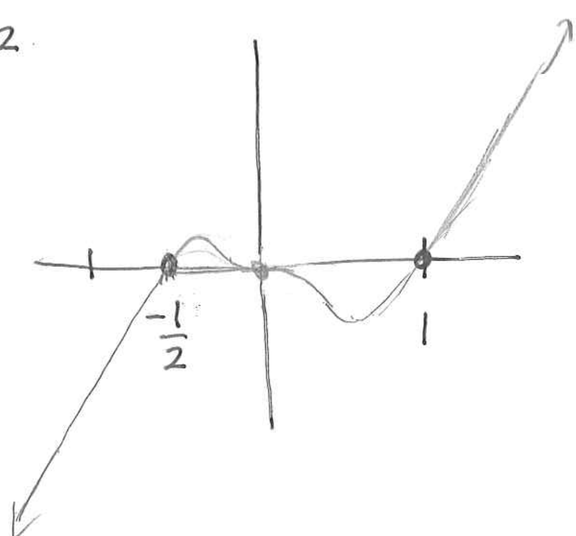
$x = 0$ — mult 3

$x = 1$ — mult 1

$x = -1/2$ — mult 1

yint: $(0,0)$

lead term: $2x^5$



Example: Sketch a graph of the polynomial. $C > A > 1 > B > 0$

ⓐ $y = x^2 + (A+1)x + A$

xint: $0 = x^2 + (A+1)x + A$

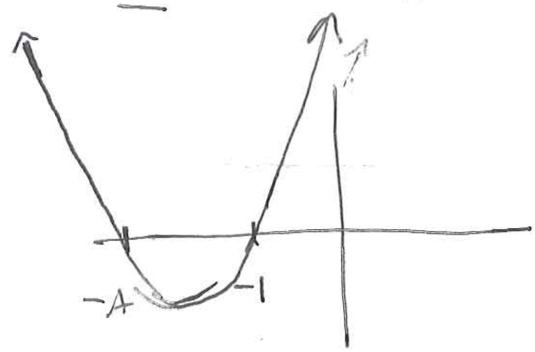
$= (x+1)(x+A)$

$x = -1$, $x = -A$

mult 1   mult 1

y-int: $(0, A)$

L.T.: x^2



ⓑ $y = -2(x-A)(x+B)^2(x+C)^3$

$C > A > 1 > B > 0$

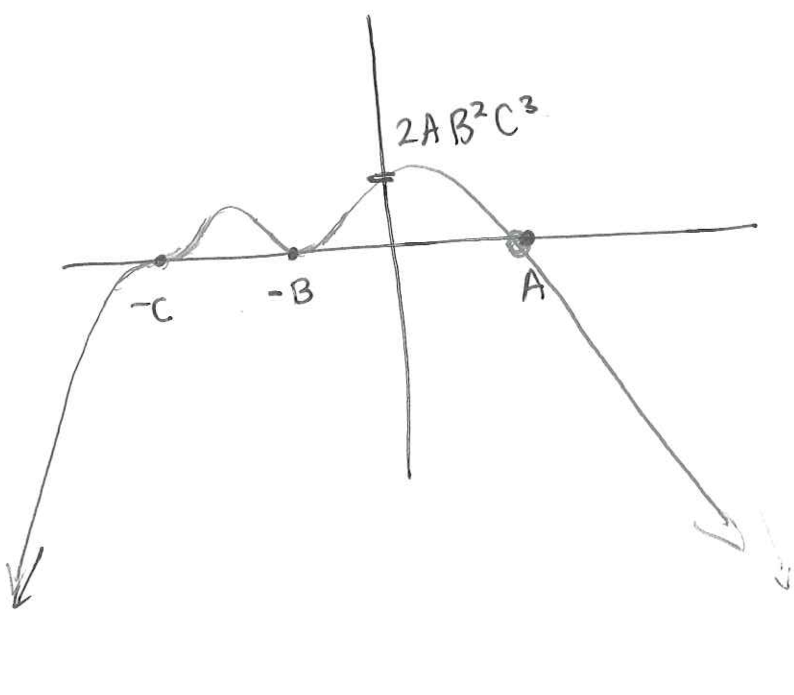
xint: $x = A$, $x = -B$, $x = -C$

mult 1   mult 2   mult 3

yint: $-2(-A)(B)^2(C)^3$

$= 2AB^2C^3$

LT: $-2(x)(x^2)(x^3) = -2x^6$

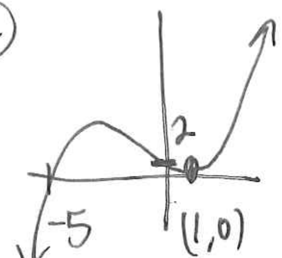


Find a formula for a polynomial:

FACTORED FORM IS USUALLY EASIEST.

Example 2: Find the formula for the poly w/ the given properties.

(a)



xint: $x = -5 \leftarrow$ mult 1

$x = 1 \leftarrow$ mult 2

$$y = a(x+5)^1 (x-1)^2$$

??

y int $(0,2) \Rightarrow \quad 2 = a(5)(-1)^2$

$$a = 2/5$$

$$\Rightarrow y = \frac{2}{5}(x+5)(x-1)^2$$

(b) degree 5 w/ x intercepts only at $(1,0)\ (-2,0)\ (4,0)$

and ~~extr.~~ end behavior as $x \to -\infty,\ y \to +\infty$

as $x \to +\infty,\ y \to -\infty$.

??

(many, many possible solutions)

$$y = -(x-1)^2(x+2)^2(x-4)^1 \quad \text{or} \quad -(x-1)^3(x+2)(x-4)$$

downward S shape

Factoring techniques for polynomials:

DO NOT DO: ~~Descartes Rule of sign pg 275~~) 3.3.
(OMIT) ~~The upper/Lower bounds thm pg 276~~

You will need to know : Rational zeros theorem:

$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$

~~Say that p, q~~ the only rational zeros of f(x) must be of the form

$\frac{p}{q}$ where p is a factor of a_0

q is a factor of a_n.

Example: Find all rational zeros. of

$f(x) = 12x^4 + 2x - 6$

q					p
	12	1	1	6	
	3	4	2	3	
	6	2			

possible rational zeros:

$\frac{6}{1}, \frac{6}{12}, \frac{1}{1}, \frac{1}{12}$

$\frac{6}{4}, \frac{6}{3}, \frac{1}{4}, \frac{1}{3}$

$\frac{6}{6}, \frac{6}{2}, \frac{1}{6}, \frac{1}{2}$

Factor Thm: For a polynomial:

$x = c$ is a zero $\Longleftrightarrow$ $x - c$ is a factor

$\Updownarrow$

$(c, 0)$ is an x intercept

Remainder theorem: For a polynomial $p(x)$, if we divide $p(x)$ by $x - c$, the remainder from long division is $p(c)$.

Example: Factor completely:

(a) $p(x) = \underline{x^3 + 4x^2 + 3x - 2}$ ← poly.

$x = -2$ is a zero from calc. $\longrightarrow$ $(x+2)$ is a factor.

$$
\begin{array}{r l}
 & x^2 + 2x - 1 \\
x+2 \,\big) & x^3 + 4x^2 + 3x - 2 \\
 & -(x^3 + 2x^2) \\
 & \quad 2x^2 + 3x - 2 \\
 & \quad -(2x^2 + 4x) \\
 & \qquad -x - 2 \\
 & \qquad -(-x - 2) \\
 & \qquad\quad 0
\end{array}
$$

$\Longrightarrow p(x) = (x+2)(\underline{x^2 + 2x - 1})$

↑

zeros $0 = x^2 + 2x - 1$

Q.F.

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$x = -1 \pm \sqrt{2}$$

$x = -1 + \sqrt{2}$

$x = -1 - \sqrt{2}$

$p(x) = (x+2)(x - (-1+\sqrt{2}))(x - (-1-\sqrt{2}))$

Ex. Find the real zeros.

$y = x^3 - 5x^2 + 2x + 12$

$x = 3$ is a zero $\Rightarrow$ $\underline{\underline{x - 3}}$ is a factor.

zero

$$\begin{array}{r|rrrr} 3 & 1 & -5 & 2 & 12 \\ & \downarrow & 3 & -6 & -12 \\ \hline & 1 & -2 & -4 & \boxed{0} \end{array}$$

$\leftarrow$ coefficients of poly

mult by zero to go diagonally, add going down.

coeff $\rightarrow$ of quotient

$\uparrow$ remainder

$\Rightarrow x^3 - 5x^2 + 2x + 12 = \underline{\underline{(x-3)}}(\underline{x^2 - 2x - 4})$

$\uparrow$ doesn't factor

$0 = x^2 - 2x - 4$

Q.F. $\dfrac{2 \pm \sqrt{\overset{20}{4 + 16}}}{2} = 1 \pm \sqrt{5}$

$$\boxed{x = 3,\ 1 + \sqrt{5},\ 1 - \sqrt{5}}$$

You try: $y = x^4 - 7x^3 + 14x^2 - 3x - 9 = (x-3)^2(x^2 - x - 1)$

$x = 3$ (mult 2) $\Rightarrow (x-3)^2$ is a factor

Q.F. $\dfrac{1 \pm \sqrt{5}}{2}$