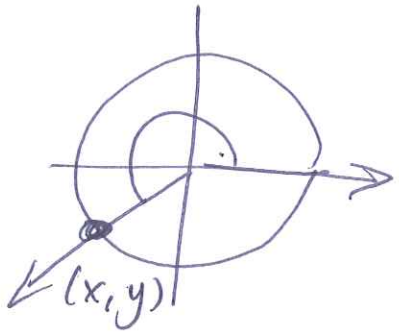
3/28/2011 (M)
Section 12

No trig formulas provided on test.
Wednesday: Review; 5.1
Thursday: Exam 3
Extra office hours; Review posted.

§6-3 continue

Recall for any angle $\theta$ in standard position defined

$\sin\theta = y$ (vertical)
$\cos\theta = x$ (horiz)
$\tan\theta = y/x$ (slope)




Domain:

$\sin\theta$: $(-\infty, \infty)$

$\cos\theta$: $(-\infty, \infty)$

$\tan\theta$: all angles except those terminate through $(0,1)$ or $(0,-1)$

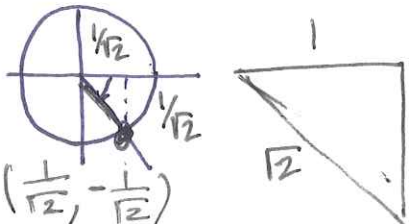
$\theta \neq \frac{\pi}{2} + 2\pi n$, where $n$ is an integer.

$\frac{3\pi}{2} + 2\pi n$

| S | A |
|---|---|
| Sin pos. | all positive |
| T | C |
| tan pos | cos pos. |

Example 1: Evaluate the trig functions $\sin\theta, \cos\theta, \tan\theta$ for

(a) $\theta = -\pi/4$.



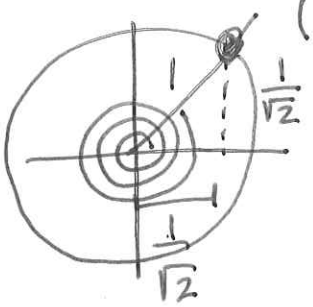

$\sin\theta = -\frac{1}{\sqrt{2}}$
$\cos\theta = \frac{1}{\sqrt{2}}$
$\tan\theta = -1$

**Def** The reference angle to $\theta$ is the acute angle formed w/ the horizontal axis.

| n | $\theta$ |
|---|---|
| 1 | 360 |
| 2 | 720 |
| 3 | 1080 |
| 4 | 1440 |

(b) (Find the ref angle first).

$\theta = 1125° = 3(360°) + 45°$



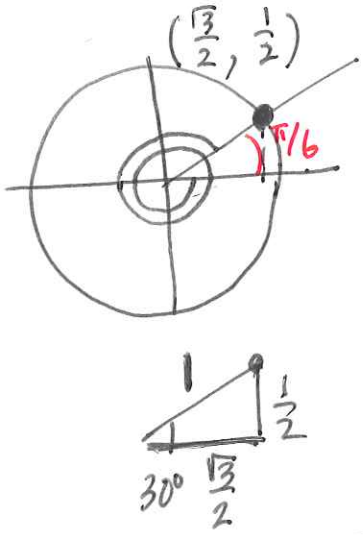

$\sin\theta = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta = 1$

(c) $\theta = -\frac{23\pi}{6} = \frac{-3\pi - \frac{5\pi}{6}}{\text{//}}$

$\hookrightarrow \frac{-18\pi - 5\pi}{6}$




$\sin\theta = \frac{1}{2}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\tan\theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

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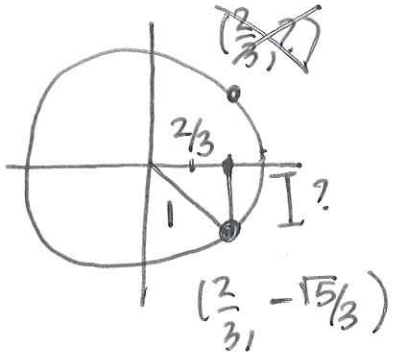
Example 2:

(a) Evaluate $\tan\theta$ if $\cos\theta = 2/3$ and $\theta$ is in Q IV.




$\left(\frac{2}{3}\right)^2 + y^2 = 1$

$y^2 = 1 - \frac{4}{9}$

$y = \pm\sqrt{5}/3$

$\tan\theta = \frac{-\sqrt{5}/3}{2/3} = \left(-\frac{\sqrt{5}}{2}\right)$

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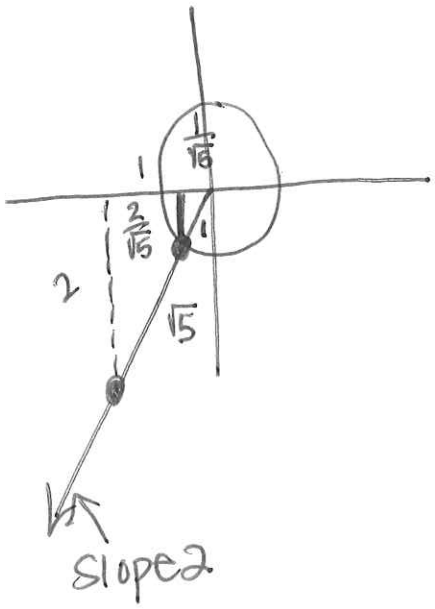
(b) Evaluate $\cos\theta$ if $\tan\theta = 2$ and $\pi \le \theta \le 3\pi/2$.




$\cos\theta = -\frac{1}{\sqrt{5}}$

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(c) Rewrite $\sec\theta$ in terms of $\sin\theta$ in QII.

"Easy" $\sec\theta = \frac{1}{\cos\theta} = \left(\frac{1}{-\sqrt{1-\sin^2\theta}}\right)$

$\cos\theta$ ↑ $\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos\theta = \pm\sqrt{1-\sin^2\theta}$

$\Rightarrow$ since x-vals are neg in QII

$\cos\theta = -\sqrt{1-\sin^2\theta}$

(d) Rewrite $\csc\theta$ in terms of $\tan\theta$ in QIII.

"Easy" = $\csc\theta = \frac{1}{\sin\theta}$

$$\tan^2\theta + 1 = \sec^2\theta = \frac{1}{\cos^2\theta} = \frac{1}{1-\sin^2\theta}$$

↑ $\cos^2\theta + \sin^2\theta = 1$

$$\tan^2\theta + 1 = \frac{1}{1-\sin^2\theta}$$

$$\frac{1}{\tan^2\theta+1} = 1-\sin^2\theta$$

$$\sin^2\theta = 1 - \frac{1}{\tan^2\theta+1}$$

$$\sin\theta = -\sqrt{1 - \frac{1}{\tan^2\theta+1}}$$

$$\Rightarrow \csc\theta = \frac{1}{-\sqrt{1-\frac{1}{\tan^2\theta+1}}}$$

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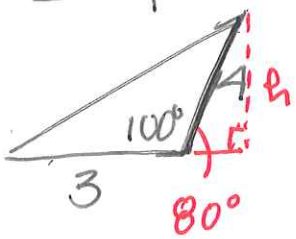
Formula for area of triangle w/angle $\theta$:

example: Find the area:



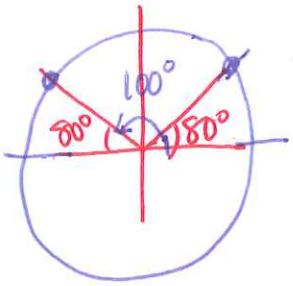

$\sin 80° = \frac{h}{4}$

$A = \frac{1}{2}(\text{base})(\text{height})$

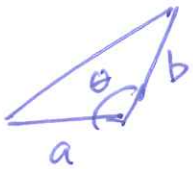
$= \frac{1}{2}(3)(h)$

$= \frac{1}{2}(3)\,4\sin 80°$

$= \frac{1}{2}(3)(4)\sin(100°)$



General Formula :




$$A = \frac{1}{2}ab\sin\theta$$