

- Exam 2 info on website; additional office hours | 2/28/2011 Section 12
- Review Solutions post tomorrow
- Wednesday - Bring questions!

§3.6 Rational Functions: $\frac{n(x)}{d(x)}$; $n(x), d(x)$ are polys.

Last time

Domain: Solve $d(x) \neq 0$.

↳ VA: zeros of reduced denom

↳ hole: common factors of $n(x), d(x)$.

x-int/zeros: Solve $n(x) = 0$.

$$\frac{n(x)}{d(x)} = 0$$

$$n(x) = 0$$

y-int: Set $x=0$ and find y

End-behavior: $\frac{n(x)}{d(x)}$ has the same end behavior as its quotient $q(x)$.

Determine the end behavior (and find any end beh asymptotes)

for:

(a) $y = \frac{1}{x}$

→ (a)

Find quotient (Long divide)

$$x \overline{) \begin{array}{r} 0 \leftarrow \text{quotient} \\ 1 \\ 0 \\ \hline 1 \leftarrow \text{remainder} \end{array}}$$

$$\frac{1}{x} = 0 + \frac{1}{x}$$

↓ as $x \rightarrow \pm\infty$
0

(b) $y = \frac{2x+1}{x+5}$

→ AS $x \rightarrow \pm\infty$, $y \rightarrow 0$
HA @ $y=0$

(c) $y = \frac{x^2+2x+1}{3x+2}$

Example: Find all asymptotes for end behavior.

(a) $y = \frac{x^2 + x}{5x^5 - 2} \Rightarrow \begin{matrix} \text{deg top} = 2 \\ \text{deg bot} = 5 \end{matrix}$
 $\Rightarrow \boxed{\text{HA @ } y = 0}$

(b) $y = \frac{2x^4 + 5x^3 + 2x + 5}{-6x^4 + 5x^2 + 9} \Rightarrow \text{deg top} = 4 = \text{deg bottom}$
 $\Rightarrow \boxed{\text{HA @ } y = \frac{2}{-6} = -\frac{1}{3}}$

(c) $y = \frac{x^2 - 2x}{x - 3} = x + 1 + \frac{3}{x - 3}$

Synthetic: $\begin{array}{r|rrrr} 3 & 1 & -2 & 0 & \\ & & 3 & 3 & \\ \hline & 1 & 1 & 3 & \end{array}$ $\Rightarrow \boxed{\text{SA @ } y = x + 1}$
↑
r(x)

(d) $y = \frac{x^3 - 2x - 5}{x + 1} = x^2 - x - 1 + \frac{-4}{x + 1}$

$\begin{array}{r|rrrr} -1 & 1 & 0 & -2 & -5 \\ & & -1 & 1 & 1 \\ \hline & 1 & -1 & -1 & -4 \end{array}$ $\Rightarrow \text{nonlinear asympt.}$
 $y = x^2 - x - 1.$
↑
r(x)

Example: Graph.

(a) $y = \frac{(x+1)(x-2)}{(x-1)(x+3)}$

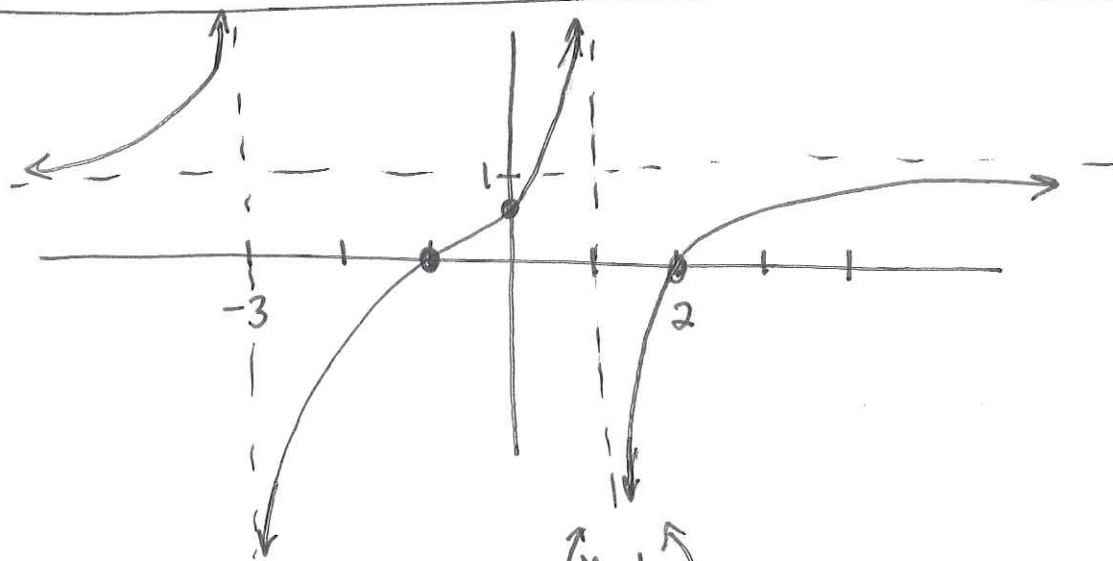
Domain: $x \neq 1, x \neq -3$
 $\uparrow \qquad \qquad \uparrow$
 VA @ $x=1, x=-3$

xint: numerator
 $x = -1, 2$

yint: plug in $x=0$
 $\frac{(1)(-2)}{(-1)(3)} = \frac{2}{3}$

End behavior: deg $n=2$ deg $d=2$

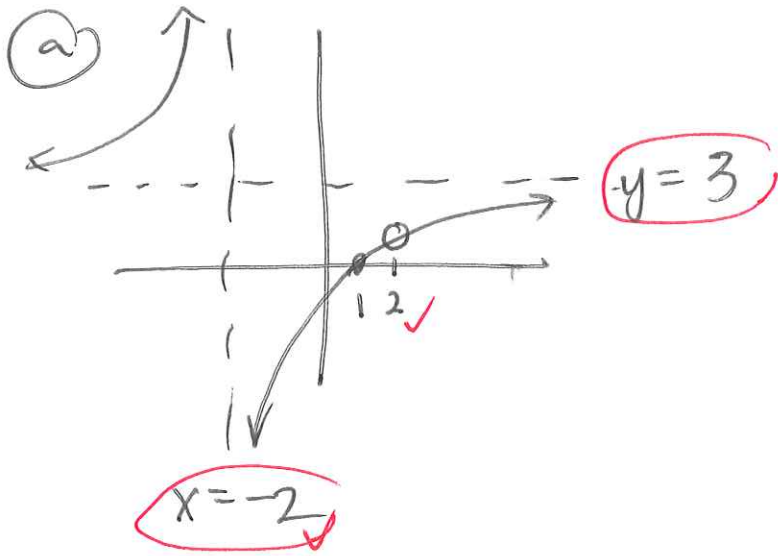
$\Rightarrow \frac{\text{lead coef } n}{\text{lead coef } d} = \frac{1}{1} \Rightarrow \text{HA @ } y=1$



$y = \frac{(x+1)(x-2)}{(x-1)(x+3)}$

$x = -3$		$x = 1$	
-3.01	-2.99	$x = 0.99$	$x = 1.01$
$\frac{(-)(-)}{(-)(-)}$	$\frac{(-)(-)}{(-)(+)}$	$\frac{(+)(-)}{(-)(+)}$	$\frac{(+)(-)}{(+)(+)}$
"	"	"	"
+	-	-	+

Example: Find a formula for the rational function:



Domain: $x \neq -2, 2$
 \uparrow \uparrow
 VA hole

VA @ $x = -2 \Rightarrow x+2$ must be in denom

hole @ $x = 2 \Rightarrow x-2$ must be in denom AND num.

int @ $x = 1 \Rightarrow x-1$ must be in numerator.

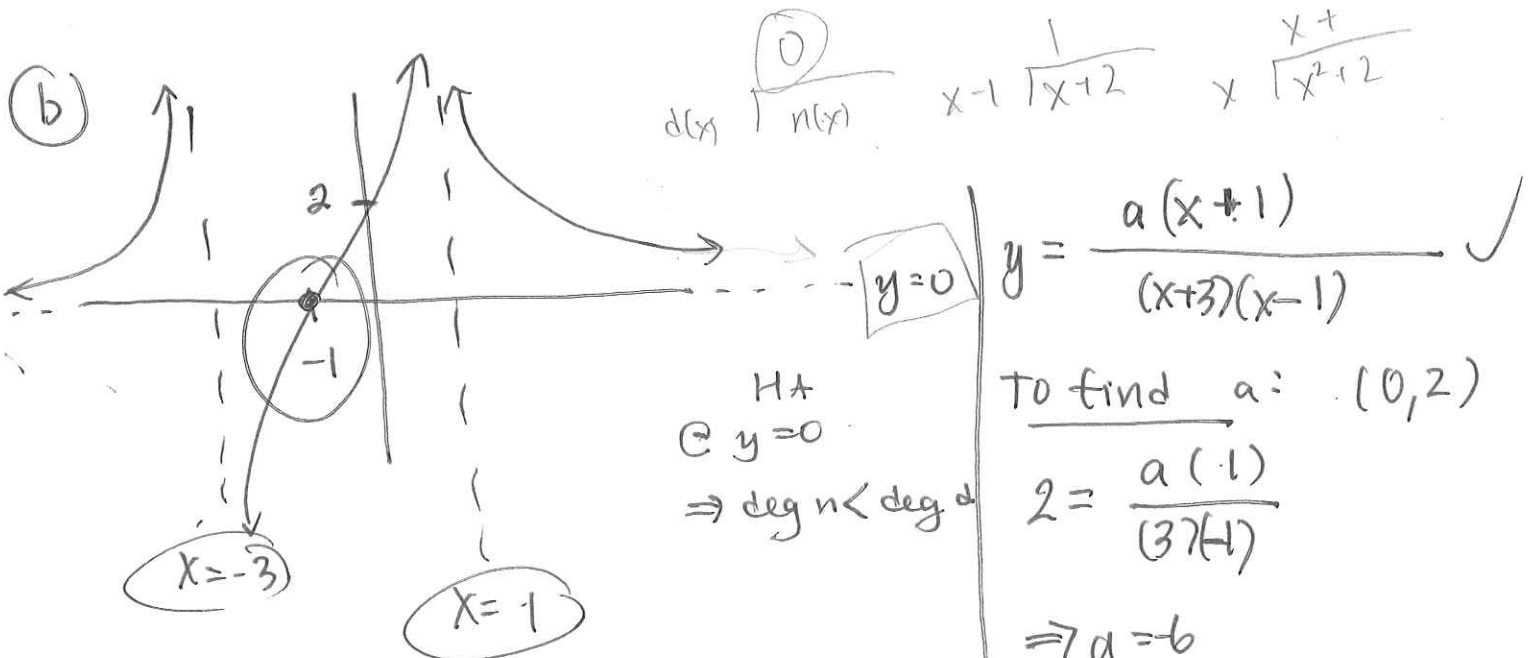
HA @ $y = 3$
 $\Rightarrow \deg \text{top} = \deg \text{d}$

$$3 = \frac{\text{lead coef of } n}{\text{lead coef of } d}$$

$$\Rightarrow \frac{3(x-1)(x-2)}{(x+2)(x-2)}$$

$\swarrow \deg^2$
 $\searrow \deg^2$

$$\begin{array}{r} 0 \\ x^2+2x+3 \overline{) x^2+1} \end{array}$$



$$d(x) \overline{) n(x)} \quad x-1 \overline{) 1} \quad x \overline{) x+2}$$

$$y = \frac{a(x+1)}{(x+3)(x-1)}$$

To find a : $(0, 2)$

$$2 = \frac{a(-1)}{(3)(-1)}$$

$$\Rightarrow a = -6$$

$$\Rightarrow y = \frac{-6(x+1)}{(x+3)(x-1)}$$