Wednesday 2.7

~~2/2011~~
2/21/2011
Section 1d

$\boxed{\S 2.7}$ Inverse functions:

↳ Idea: Reversing/undoing f.

Example 1: Given f(x) below find the inverse function, if it exists

(a)

x	0	1	2
$f(x)$	2	4	6

← outputs of f(x)

Inverses:

x	2	4	6
y	0	1	2

inputs of inverse

(b) $f(x) = x+5$

inverse:

$x-5$.

Notation: We'll denote the inverse of f(x) by $f^{-1}(x)$.

-1 is NOT an exponent.

i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$

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in general,

given a table for f(x), we simply reverse the role of input/output to find the inverse.

Caution: not every table/function has an inverse function

Example: FUNCTIONS THAT DO NOT have an inverse.

(a)

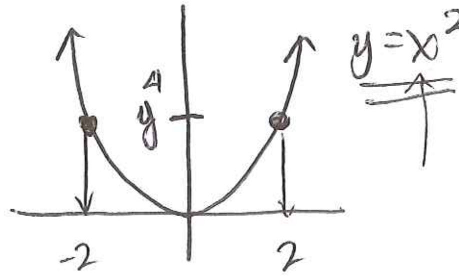
x	0	1	2	3
f(x) = y	0	1	0	2

x	0	1	0	2
y ~~f(x)~~	0	1	2	3

the flipped table is not a function. x=0, 2 solutions

f(x) does not have an inverse function

(b)



inverse: formula/graph that gives the original input when output specified

$$\pm\sqrt{y} = x$$

two solutions!

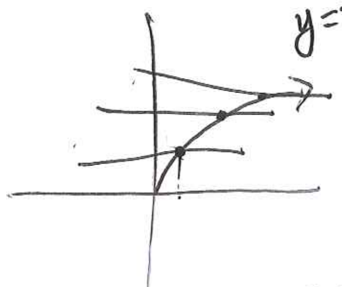
For a function to have an inverse:

- Each output (comes from) has at most one input. ← "ONE TO ONE"
- Equivalently, must pass the Horizontal line test:

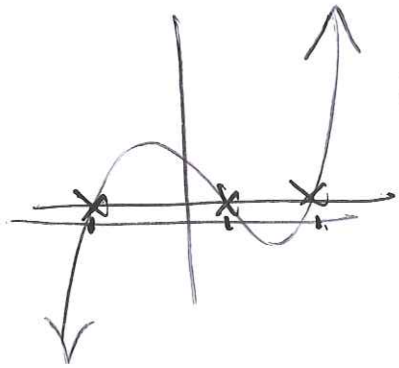
"For each horizontal line, the line and graph intersect in @ most one point."

Example: Determine if inverse exists. If so, sketch the inverse:

(a)



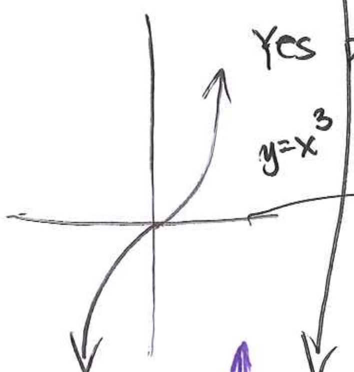
(b)



NO fails HLT

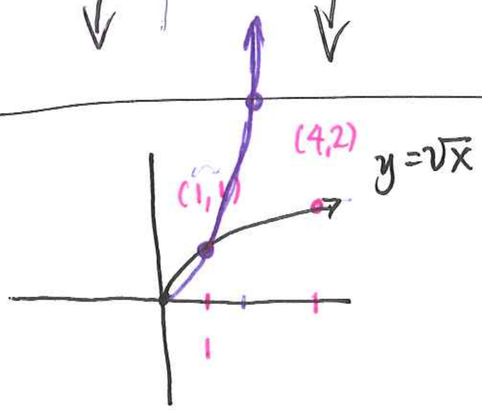
Yes has an inverse (see sketch below)

(c)

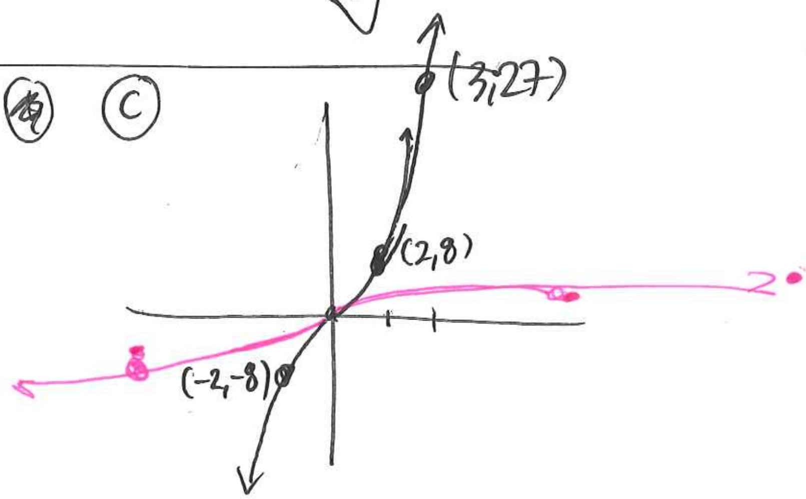


Yes passes HLT

(a)



(c)



Example: Given the following 1-1 functions, find the inverse.

(a) $f(x) = \dfrac{1}{x+2} \longrightarrow y = \dfrac{1}{x+2}$

switch variables and solve for y:

$$x = \frac{1}{y+2}$$

$$x = \frac{1}{y+2}$$

$$\frac{1}{x} = y + 2$$

$$\boxed{\frac{1}{x} - 2 = y}$$ ← inverse function : $f^{-1}(x) = \frac{1}{x} - 2.$

(b) $f(x) = (x+2)^3 - 1$

$$y = (x+2)^3 - 1$$

switch variables :

$$x = (y+2)^3 - 1$$

$$\sqrt[3]{x+1} - 2 = y$$ ← inverse: $f^{-1}(x) = \sqrt[3]{x+1} - 2$

(c) Graph part (b) $f(x)$ and $f^{-1}(x)$.

In general, $y = f^{-1}(x)$ is the reflection of $y=f(x)$ over the line $y=x$.

Warning! For application problems, do not switch variables.

Example: let F = farenheit temp

C = ~~Cle~~ Celcius temp.

$$C = \frac{5}{9}(F-32)$$

input: Farenheit temp.
output: celcius temp

Find a formula for the inverse and explain its application.

Inverse:

Input: Celcius temp
output: Farenheit temp

Solve for F

$$C = \frac{5}{9}(F-32)$$

$$\frac{9}{5}C = F - 32$$

$$\boxed{\frac{9}{5}C + 32 = F}$$

This equation allows us to convert °C → °F.

Example: Suppose a chemist mixes 30% saline solution w/ 50% saline solution. If she has 100mL of 50% solution,

X mL of 30% and

the concentration is

$$\frac{0.3x + 50}{x + 100} = C$$

Input: x mL of 30%
output: C, concentration

Find the inverse; explain its application.

Inverse:

input: concentration

output: x mL of 30%

$$\frac{0.3x+50}{x+100} = C$$

Solve for x:

$0.3x + 50 = (C)(x+100)$

$0.3x + 50 = Cx + 100C$

$0.3x - Cx = 100C - 50$

$x(0.3 - C) = 100C - 50$

$$\boxed{x = \frac{100C-50}{0.3-C}}$$

Given the concentration, the chemist can determine x mL, the amt of 30% solution to be added to produce concentration C.

Domain of original = Range of inverse

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Tables: switch input/output vals.

Graphs: reflect over $y = x$

Alg. expressions (non-app): set $f(x) = y$, switch var, solve for y. this is $f^{-1}(x)$.

Applications: solve for input.