§2.6

* I do things slightly different than the book.

Read text! (Before next class)

Short Quiz - open notes! (No Book)

Warmup:

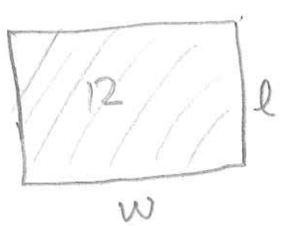
(a) Express the area (A output) of a rectangle whose width is twice its length in terms of its width (input w).

width ↑ big

$A = lw = \boxed{\frac{1}{2}w^2 = A}$

Elim l: $w = 2l \rightarrow \frac{1}{2}w = l$

(b) Express the perimeter (output) of a rectangle whose area is 12 in terms of its length (input).



$P = 2l + 2w = 2l + 2\left(\frac{12}{l}\right) = \boxed{2l + \frac{24}{l} = P}$

Elim w: $12 = lw \rightarrow \frac{12}{l} = w$

In general:

0.) Identify the desired input/output.

1.) Draw a picture (if appropriate) and label variables.

2.) Write the "easy" equation for the output

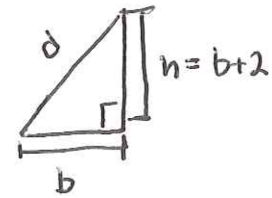
3.) Eliminate variables if neccessary (using the constraint if given) then solve for the desired output (if neccessary).

4.) Find the feasible/reasonable domain.

Example 1: Suppose a triangle w/ a right angle is 2 units taller than its base.

Find a formula for its

(a) diagonal (output) in terms of its base (input).



$$d^2 = b^2 + (b+2)^2$$

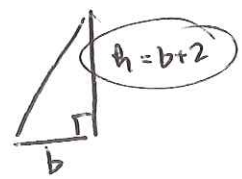
$$d^2 = b^2 + b^2 + 4b + 4$$

$$d^2 = 2b^2 + 4b + 4$$

$$d = \sqrt{2b^2+4b+4}$$

$0 < b$

(b) Area (output) in terms of its height (input). h



$$A = \frac{1}{2} b h = \boxed{\frac{1}{2}(h-2)h = A}$$

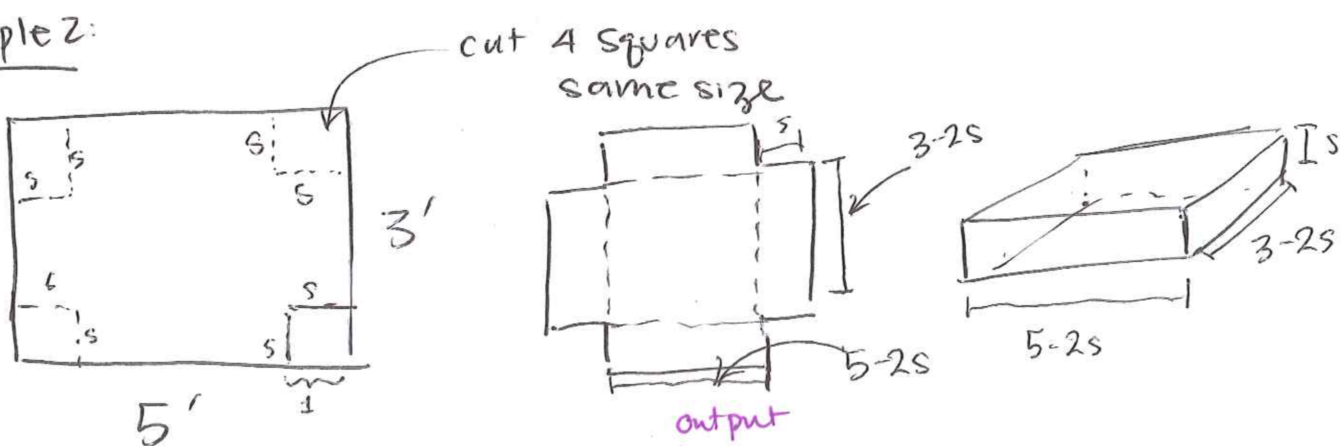
eliminate: b

CONSTRAINT: $h = b+2$

$h - 2 = b$

$2 < h$

Example 2:



Find a formula for volume (output) in terms of s (input), the side length of one of the squares cut out.

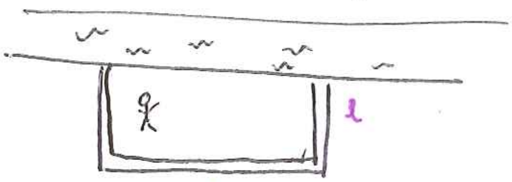
$$V = (5-2s)(3-2s)(s)$$

D: $0 < s < 1.5$

Graph of V on $0 < s < 1.5$

Example 3:

Fencing a ~~garden~~ pasture along a very straight long river. Only need 3 sides of fencing.



(a) Find a formula for area if we have 500 ft of fencing. (in terms of either l or w).

$A = lw = (250 - \frac{w}{2})(w)$ ← width in ft

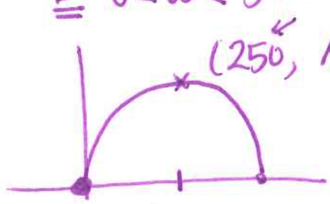
↑ constraint:

$500 = 2l + w \rightsquigarrow 250 - \frac{w}{2} = l$

(b) Find the maximum area that can be enclosed.

D: $0 < w < 500$

$(250, A(250)) = (250, 31{,}250)$



max area

$A(250) = (250 - \frac{250}{2})(250)$

$= 31{,}250$

→ If the width is 250 ft, the area is the maximum value of $31{,}250 \text{ ft}^2$.

Example 4: Suppose you are making a can (cylinder) that must hold 120 cubic cms of fluid. If the top and bottom costs \$0.1/cm² and the lateral surface area material costs \$0.30/cm², find the minimum cost.