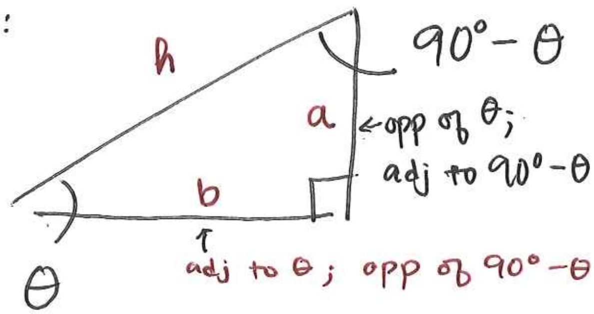
4/18/2011 M
Section 12

- Grade reports post to D2L tomorrow night
- Exam 4 th April 28th (Review posts this thursday)

Warmup:



Rewrite in terms of $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\csc\theta$, $\sec\theta$:

$$\left.\begin{aligned}
\sin(90°-\theta) &= \frac{b}{h} = \cos(\theta)\\
\cos(90°-\theta) &= \frac{a}{h} = \sin(\theta)\\
\tan(90°-\theta) &= \frac{b}{a} = \cot(\theta)\\
\sec(90°-\theta) &= \frac{h}{a} = \csc(\theta)\\
\csc(90°-\theta) &= \frac{h}{b} = \sec(\theta)\\
\cot(90°-\theta) &= \frac{a}{b} = \tan(\theta)
\end{aligned}\right\}$$ Cofunction Identities

P 528 List of all Fundamental Identities (know these!)

Reciprocal: $\sec\theta = \frac{1}{\cos\theta}$; $\csc\theta = \frac{1}{\sin\theta}$; $\cot\theta = \frac{1}{\tan\theta}$

Quotient: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean:

$$\sin^2\theta + \cos^2\theta = 1$$

$\div \cos^2\theta$

$$\tan^2\theta + 1 = \sec^2\theta$$

$\div \sin^2\theta$

$$1 + \cot^2\theta = \csc^2\theta$$

neg-angle identities :

$\sin(-t) = -\sin t$

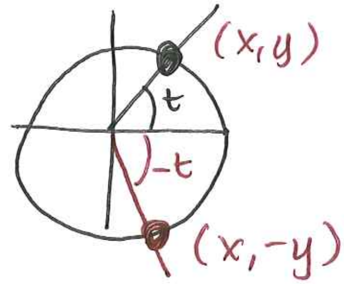
$\cos(-t) = \cos(t)$

$\tan(-t) = -\tan(t)$

$\sec(-t) = \sec(t)$

$\csc(-t) = -\csc(t)$

$\cot(-t) = -\cot(t)$



Practice w/ simplifying:

1.) $$\frac{\sin x \sec x}{\tan x} = \frac{\sin x \cdot \frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = 1$$

2.) $$\frac{1+\cot(-A)}{\csc(-A)} = \frac{1+\frac{\cos(-A)}{\sin(-A)}}{\frac{1}{\sin(-A)}} = \frac{1+\frac{\cos(A)}{-\sin(A)}}{\frac{1}{-\sin A}}$$

$$= \left(1+\frac{\cos A}{-\sin A}\right)(-\sin A) = \boxed{-\sin A + \cos A}$$

3.) $$\frac{\tan x \cos x \csc x}{1+\cot^2 x} = \frac{\tan x \cos x \cancel{\csc x}}{\csc^{\cancel{2}} x} = \frac{\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}}{\frac{1}{\sin x}} = \boxed{\sin^2 x}$$

$\sin^2 x + \cos^2 x = 1$

4.) $$\frac{\cos x}{\sec x + \tan x} = \frac{(\cos x)}{\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right)} \cdot \frac{\cos x}{\cos x} = \frac{\cos^2 x}{1+\sin x}$$

$$\boxed{1-\sin x} = \frac{(1+\cancel{\sin x})(1-\sin x)}{1+\cancel{\sin x}} = \frac{1-\sin^2 x}{1+\sin x}$$

Practice w/ identities:

Warning:

Solving an equation $\neq$ Proving an identity.

Solving an equation ↑ only true for some (maybe no) values.

Proving an identity ↑ Equation that is always true

e.g. $x^2 = 4$

$x = \pm 2$

equation

$\sqrt{x^2} = |x|$

↑ always true

$\Rightarrow$ identity.

Prove the identity: Pick one side, simplify until you get the other side.

1.) $\frac{\tan x}{\sec x} = \sin x$

$$\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{1} = \sin x \checkmark$$

2.) $(1-\cos\beta)(1+\cos\beta) = \frac{1}{\csc^2\beta}$

You try:

3.) $(\tan y + \cot y) \sin y \cos y = 1$

4.) $\dfrac{\sec x + \csc x}{\tan x + \cot x} = \sin x + \cos x.$

3.) $\cancel{\tan y}$ $\left(\dfrac{\sin y}{\cos y} + \dfrac{\cos y}{\sin y}\right) \sin y \cos y$

$\sin^2 y + \cos^2 y = 1$

4.) $\dfrac{\left(\dfrac{1}{\cos x} + \dfrac{1}{\sin x}\right)(\sin x \cos x)}{\left(\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x}\right)(\sin x \cos x)} = \dfrac{\sin x + \cos x}{\sin^2 x + \cos^2 x}$

$= \boxed{\sin x + \cos x}$

$\boxed{\S 7.3}$ Memorize (p 542)

$\boxed{\text{Double angle formulas}}$:

$\boxed{\sin(2x) = 2\sin x \cos x.}$

$\boxed{\cos(2x) = \cos^2 x - \sin^2 x}$

$= (1 - \sin^2 x) - \sin^2 x$

$\boxed{\cos(2x) = 1 - 2\sin^2 x}$

$= \cos^2 x - (1 - \cos^2 x)$

$\boxed{\cos(2x) = 2\cos^2 x - 1}$

Pyth.

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin^2 x = 1 - \cos^2 x$