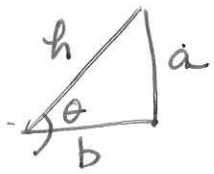
3/25/2011
Section 12

§6.2 Right triangle trig



SOH CAH TOA $0 < \theta < 90°$

$\sin\theta = \frac{a}{h}$	$\csc\theta = \frac{h}{a}$
$\cos\theta = \frac{b}{h}$	$\sec\theta = \frac{h}{b}$
$\tan\theta = \frac{a}{b}$	$\cot\theta = \frac{b}{a}$

Definitions

Reciprocal:

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

Quotient:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Pythagorean:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Identity:

Scratch work:

$$a^2 + b^2 = h^2$$

$$\frac{h^2} \quad \frac{h^2}$$

$$\frac{a^2}{h^2} + \frac{b^2}{h^2} = 1$$

$$\left(\frac{a}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = 1$$

$$\sin^2\theta + \cos^2\theta = 1$$

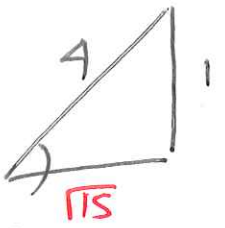
or $\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{h^2}{b^2}$

$$\tan^2\theta + 1 = \sec^2\theta$$

Example: Find all trig ratios given the following

Method: Draw a picture

(a) $\sin\theta = \frac{1}{4}$



$\cos\theta = \frac{\sqrt{15}}{4}$

$\tan\theta = \frac{1}{\sqrt{15}}$

$\csc\theta = 4$

$\sec\theta = 4/\sqrt{15}$

$\cot\theta = \sqrt{15}$.

Method: Using identities

$\sin\theta = 1/4$

$\Rightarrow \csc\theta = 4$

$\sin^2\theta + \cos^2\theta = 1$

$\frac{1}{16} + \cos^2\theta = 1$

$\cos^2\theta = 15/16$

$\cos\theta = \sqrt{15}/4$

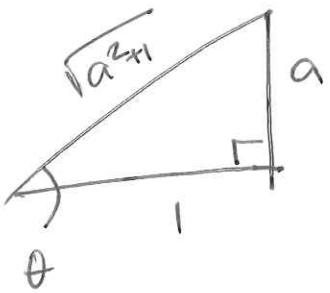
$\sec\theta = \frac{1}{\cos\theta} = 4/\sqrt{15}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{4}\,4}{\frac{\sqrt{15}}{4}\,4} = \frac{1}{\sqrt{15}}$

$\cot\theta = \sqrt{15}$

$\sin\theta = 1/4$ ✓
$\cos\theta$ ✓
$\tan\theta$ ✓
$\csc\theta$ ✓
$\sec\theta$ ✓
$\cot\theta$ ✓

(b) $\tan\theta = a$, $a > 0$ constant



SOH CAH TOA and flip...

$\tan\theta = a$ "

$\Rightarrow \cot\theta = \frac{1}{a}$

$\tan^2\theta + 1 = \sec^2\theta$

$a^2 + 1 = \sec^2\theta$

$\sqrt{a^2+1} = \sec\theta$

$\cos\theta = \frac{1}{\sqrt{a^2+1}}$

~~Sin~~
~~cos~~
~~tan~~
~~csc~~
~~sec~~
~~cot~~

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \frac{1}{a^2+1} = 1$$

$$\sin^2\theta = 1 - \frac{1}{a^2+1}$$

$$\sin\theta = \sqrt{\frac{a^2}{a^2+1}}$$

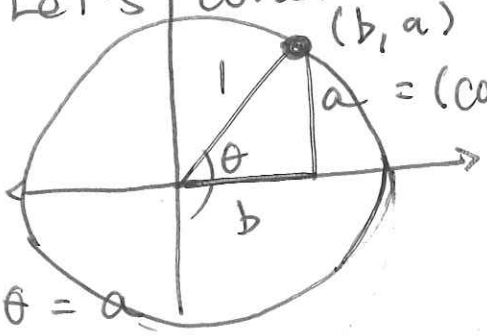
$$\sin\theta = \frac{a}{\sqrt{a^2+1}}$$

$$\csc\theta = \frac{\sqrt{a^2+1}}{a}$$

6.3 Trig function of all angles.

Idea: It doesn't matter what size triangle we work w/ when determining ratios.

Let's consider triangles of hypotenuse 1.



$\sin\theta = a$

$\cos\theta = b$

$\tan\theta = \frac{a}{b}$

Define $-\infty < \theta < \infty$

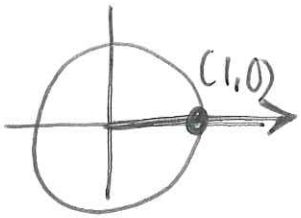
$\sin\theta$ = y-value of the intersection of unit circle and term side.

$\cos\theta$ = x value " "

$\tan\theta = \frac{y}{x}$ (slope of term side)

Evaluate sin, cos, tan of

(a) $\theta = 0°$

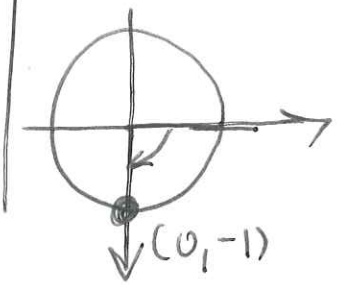


$\sin 0° = 0$

$\cos 0° = 1$

$\tan 0° = 0$

(b) $\theta = -\pi/2$

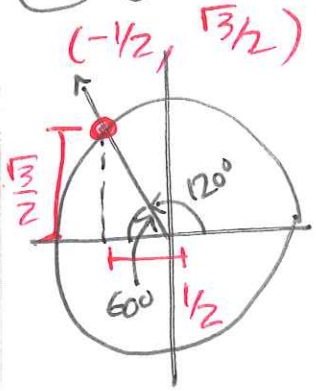


$\sin(-\pi/2) = -1$

$\cos(-\pi/2) = 0$

$\tan(-\pi/2) = DNE.$

(c) $\theta = 120°$

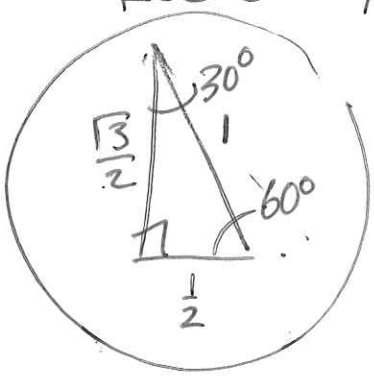


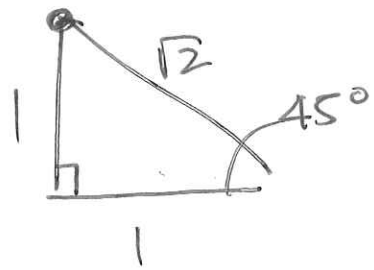
$\sin\theta = \sqrt{3}/2$

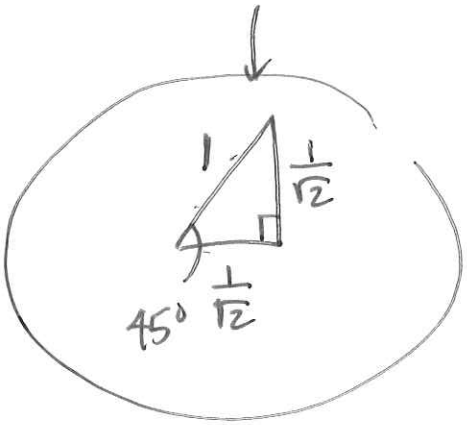
$\cos\theta = -1/2$

$\tan\theta = -\sqrt{3}$

Recall Special triangle







(d) $\theta = \frac{13\pi}{3}$

$= 4\pi + \frac{\pi}{3}$