2/18/2011
Section 12

§2.7 "Combining Functions"

Given functions f, g

$(f+g)(x) = f(x) + g(x)$
$(f-g)(x) = f(x) - g(x)$
$(f\cdot g)(x) = f(x) \cdot g(x)$
$(f/g)(x) = f(x)/g(x)$

$(f \circ g)(x)$

Given the domain of f, domain of g,
what can we conclude about the domain
of $f \pm g$, $f \cdot g$, f/g?

Example 4: Find the domain of $f+g$, $f-g$, $f\cdot g$, f/g, g/f.

(a) $f(x) = \sqrt{9-x}$

$g(x) = \dfrac{x}{x^2-1}$

Dom f: $9 - x \geq 0$, $9 \geq x$

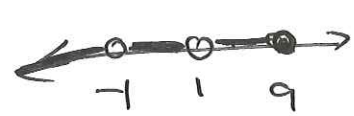
Dom g: $x^2 - 1 \neq 0$, $x \neq \pm 1$

Dom f+g:

Dom f-g:

Dom f·g:

Same!



"common values"

$(-\infty, -1) \cup (-1, 1) \cup (1, 9]$

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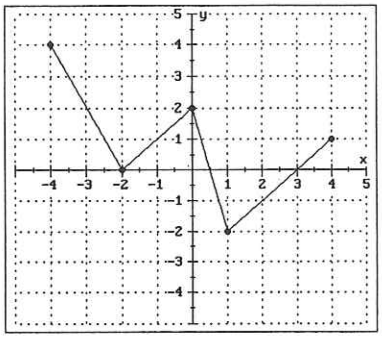
Worksheet - Combining Functions (section 1.6)

1. Use the functions $f(x)$, $g(x)$, and $h(x)$ given below to determine the following:

$f(x) = 3x^2 - 7$

x	-2	0	3	4
$g(x)$	4	-3	6	1



$h(x)$

a) $f(0) = -7$

b) $g(0) = -3$

c) $h(0)$

$= 2$

d) $g(0) + h(0) = -3 + 2 = -1$

2. Use the functions $r(x)$ and $n(x)$ below to determine the following, if possible:

x	-1	0	1	2	3
$r(x)$	2	-1	3	1	2

x	-1	0	1	2	3
$n(x)$	0	-2	2	-1	1

a) $(r+n)(-1)$

$= r(-1) + n(-1) = 2 + 0 = 2$

b) $(n-r)(0)$

$n(0) - r(0) = -2 - (-1) = -1$

c) $\left(\frac{r}{n}\right)(-1)$

$= \frac{r(-1)}{n(-1)} = \frac{2}{0}$ undefined

d) $(rn)(2)$

$r(2)n(2) = (1)(-1) = -1$

e) $\left(\frac{n}{r}\right)(-1)$

$= 0$

f) $(n(r(3))$

$= n(2) = -1$

g) $(r \circ r)(0)$

$r(\underbrace{r(0)}_{-1})$

$= r(-1)$

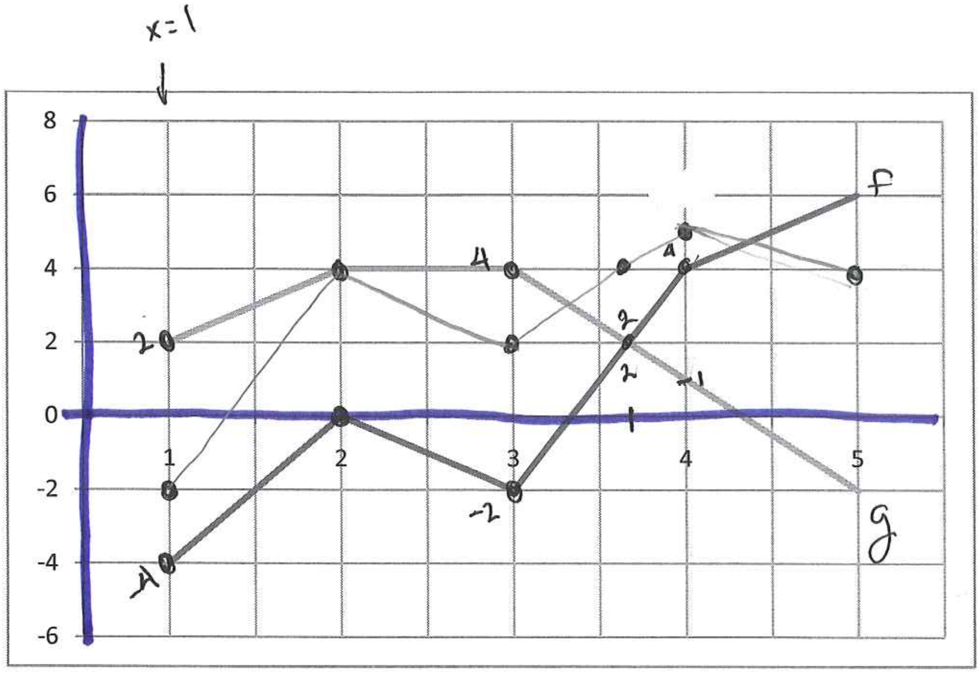
$= 2$

h) $(n \circ n)(1)$

$n(n(1))$

$= n(2)$

$= -1$



add y-values .

Sketch a graph of $(f+g)(x)$ on the axes above.

$$\shortparallel$$

$f(x)+g(x)$

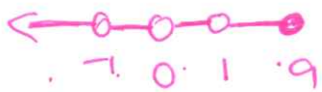
$\uparrow \quad \uparrow$

vertical values .

Dom f/g : common values in dom f, dom g (like before) without $g(x) = 0$.

$$\frac{x}{x^2-1} \neq 0$$

$$x \neq 0$$

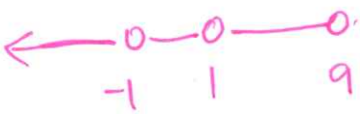


$(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, 9]$

Dom g/f : common values in dom f, dom g without $f(x) = 0$.

$$\sqrt{9-x} \neq 0$$

$$\boxed{x \neq 9}$$



$(-\infty, -1) \cup (-1, 1) \cup (1, 9)$

Composition: "Plug one function into another"

$$h(x) = f \circ g(x) = f(g(x))$$

Step 1 / inside function

Step 2 / outside function.

Example 5: Evaluate:

$f(x) = x^2+1$ $g(x) = \dfrac{x}{x+1}$

ⓐ $f \circ g(x) = f(g(x))$

$= f\left(\dfrac{x}{x+1}\right)$

$= \boxed{\left(\dfrac{x}{x+1}\right)^2 + 1}$

$\neq$

ⓑ $g \circ f(x) = g(f(x))$

$= g((x^2+1))$

$= \boxed{\dfrac{x^2+1}{x^2+2}}$

ⓒ

$$g(g(x)) = g\left(\frac{x}{x+1}\right)$$

$$= \frac{\left(\frac{x}{x+1}\right)}{\left[\left(\frac{x}{x+1}\right)+1\right]} \frac{(x+1)}{(x+1)}$$

$$= \frac{x}{x+x+1}$$

$$= \frac{x}{2x+1}$$

Decomposition:

$f \circ g(x)$ is a decomp of $h(x)$ if

$$h(x) = f(g(x)).$$

Example 5: Decompose $h(x) = f(g(x))$. $\left(\begin{array}{l} f(x) \neq x \\ g(x) \neq x. \end{array}\right)$

outside 2nd.   inside 1st

(a) $h(x) = \sqrt{\boxed{x^2} + 5}$

$g(x) = x^2$
$f(x) = \sqrt{x+5}$

OR

$g(x) = x^2 + 5$
$f(x) = \sqrt{x}$

(b) $h(x) = \left(\boxed{(x+2)}^2 - 10\right)^{4/3}$

$g(x) = (x+2)^2$
$f(x) = (x - 10)^{4/3}$

OR

$g(x) = x + 2$
$f(x) = (x^2 - 10)^{4/3}$

OR

$g(x) = (x+2)^2 - 10$
$f(x) = x^{4/3}$

OR

$g(x) = ((x+2)^2 - 10)^{1/3}$
$f(x) = x^4$

OR

$g(x) = ((x+2)^2 - 10)^4$
$f(x) = x^{1/3}$