4/22/2011
Section 12

§7.5 Inverse trig functions.

$\sin^{-1}(y) = \theta$ $\quad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ $\quad$ R

$\cos^{-1}(x) = \theta$ $\quad 0 \le \theta \le \pi$ $\quad$ top

$\tan^{-1}(x) = \theta$ $\quad -\frac{\pi}{2} < \theta < \pi/2.$ $\quad$ R

Example 1: Solve for θ exactly $0 \le \theta \le 2\pi$

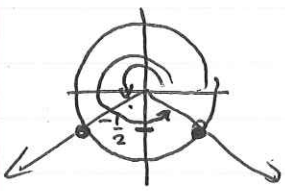
↑ counter: 1 rev.

(a) $\sin(\theta) = -1/2$

↑ $30° - 60°$

$=$

$\frac{\pi}{6}$ reference.

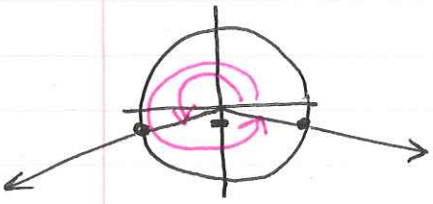


QIII: $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$

QIV: $\theta = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

(b) $\sin(\theta) = -0.25$

↑ not a special value



$\theta = \sin^{-1}(-0.25)$ $\quad$ QIV

↑ negative! -0.2527

↳ reference 0.2527

$-\sin^{-1}(0.25)$

$=$

$\sin^{-1}(0.25)$

QIII: $\pi + 0.2527 \approx 3.394$ ← Approx

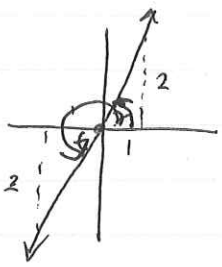
QIV: $2\pi - 0.2527 \approx 6.030$

QIII: $\pi + \sin^{-1}(0.25)$ ← Exact

QIV: $2\pi - \sin^{-1}(0.25)$

(c) $\tan(\theta) = 2$

↑ not a special value.



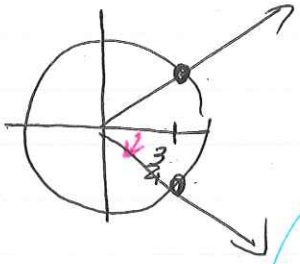
$\theta = \tan^{-1}(2)$ QI — approx: 1.017

$\theta = \pi + \tan^{-1}(2)$ QIII — $\pi + 1.017$

Example 2: Solve for θ exactly ~~$0 \le \theta \le 2\pi$~~ $-\pi \le \theta \le \pi$.

clock: ½ rev

(a) $\cos\theta = \frac{3}{4}$



$\theta = \cos^{-1}(3/4)$ QI — approx 0.7227

? QII

Counter: ½ rev.

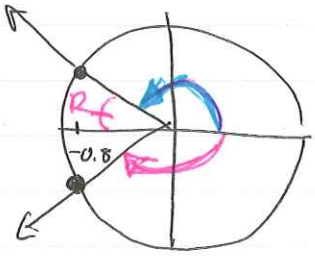
clock: $\theta = -\cos^{-1}(3/4)$ -0.7227

counter: $\theta = \cos^{-1}(3/4)$ 0.7227

approx.

(b) ~~sin θ =~~

$\cos\theta = -0.8$



Exact — approx

$\theta = \cos^{-1}(-0.8)$ QII — 2.4981

$R = \pi - 2.4981$

22

0.6435

clock: approx $-\pi + 0.6435$ — exactly $-\pi + (\pi - \cos^{-1}(-0.8)) = -\cos^{-1}(0.8)$

counter: $\pi - 0.6435$ — $\pi - (\pi - \cos^{-1}(-0.8)) = \cos^{-1}(0.8)$

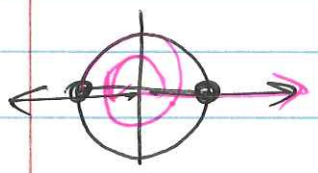
$-\pi \le \theta \le 3\pi$

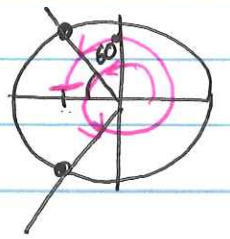
Example 3 Approximate to 3 decimals (if not a special angle)

(a) $\sin\theta + 2\sin\theta\cos\theta = 0$.

$\sin\theta(1+2\cos\theta) = 0$

$\sin\theta = 0$ OR $1+2\cos\theta = 0$





$\cos\theta = -\frac{1}{2}$

~~30~~ $60°$

$\pi/3$

Answers must be between $[-\pi, 3\pi]$

clock: $\frac{1}{2}$

counter: 1.5 revs.

Counter:

$\theta = 0, \pi, 2\pi, 3\pi$

counter

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi + \frac{2\pi}{3} = \frac{8\pi}{3}$

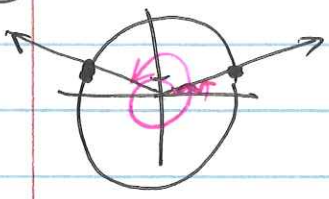
clock:

$\theta = 0, -\pi$

$\theta = -2\pi/3$

You do:

(b) $\sin\theta = 1/5$



$\theta = \sin^{-1}(1/5) \approx 0.2014$ ref: 0.2014

clock $\frac{1}{2}$ rev: none

counter 1.5 rev: $0.2014, \pi - 0.2014$

$2\pi + 0.2014, 2\pi + \pi - 0.2014$