4/15/2011
Fri Sec 10.

Over the weekend

Quotient, Recip, Pythagorean, even-odd identities

---

§ 5.5 continued

Damped harmonic motion:

$$y = ke^{-ct}\cos\omega t \quad \text{or} \quad y = ke^{-ct}\sin\omega t$$

$k$ = initial amplitude

$c$ = damping constant.

$\frac{2\pi}{\omega}$ = period.

variable amplitude: $ke^{-ct}$

---

Ex 1 Suppose a string is plucked and its oscillations/displacement follow damped harmonic motion.

If plucked (1.5cm) ($k$) and its damping constant is (0.7) ($c$) and its frequency is [0.25 cycles/sec], Find an equation for its displacement at time $t$ in seconds.

$t=0 \rightarrow$ max displacement

use cosine!

$$\Rightarrow y = 1.5e^{-0.7t}\cos\left(\frac{\pi}{2}t\right)$$

4 sec = period.

$\Rightarrow \omega = \frac{2\pi}{4}$

$\frac{2\pi}{\omega} = 4$

$2\pi = 4\omega$

$\frac{2\pi}{4} = \omega$

$\frac{\pi}{2} = \omega$

$$y = ke^{-ct}\cos\omega t$$

OR

$$ke^{-ct}\sin\omega t$$

$c$ = damping constant

$k$ = initial amp.

$2\pi/\omega$ = period.

#43. in text p 454

frequency 0.5 cycle/sec → period : 2 ⇒ $\omega = \frac{2\pi}{2} = \pi$

damping constant $c = 0.9$

Assume $k = 1$ and let $t = 0$ be the instant wind hits the building. → avg displacement → sine.

Find a formula for displacement.

$$\Rightarrow y = 1 \cdot e^{-0.9t} \sin(\pi t)$$

---

#45 in text.

3 sec later amplitude is 1/4 of the initial amplitude.

Find the damping constant.

initial amplitude : $k$

3 sec later amplitude: $\frac{1}{4}k = ke^{-c(3)}$

(1/4 kb: given; $ke^{-c(3)}$: formula)

$$\frac{1}{4} = e^{-3c}$$

$$\ln(1/4) = -3c$$

$$\frac{\ln(4)}{3} = \frac{\ln(1/4)}{-3} = c \approx 0.4621$$