

**Math 263**

**Quiz 8**

Name \_\_\_\_\_

Cervical cancer in women is most often caused by the HPV (human papillomavirus). In a randomized controlled trial on the vaccine Gardasil, one case of HPV was diagnosed among the 7897 women given the vaccine, compared to 91 cases among the 7899 given the placebo.<sup>1</sup> Is the vaccine effective? Use the following steps to decide:

(a) State the null hypothesis.

(b) State the alternate hypothesis.

(c) Find the  $z$ -value.

(d) What is the  $p$ -value?

(e) What is your conclusion about the vaccine? Give a reason for your answer.

(f) With justification, explain what you can conclude about causality in this context.

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<sup>1</sup> “Quadrivalent Human Papillomavirus Vaccine: Recommendations of the Advisory Committee on Immunization (ACIP)”, National Committee on HIV/AIDS, etc. May 2007. Reported in *Stats: Data and Models*, De Veaux et al, 3<sup>rd</sup> edn Addison Wesley 2012.

## ANSWERS

This is a two-sample hypothesis test of proportions, where the variable is having HPV. Let Population 1 be the women given the vaccine and Population 2 given the placebo.

(a) Null hypothesis: Proportion with HPV is equal in the two groups

$$H_0: p_1 = p_2$$

(b) Alternative hypothesis: Proportion with HPV is lower in the vaccine group

$$H_a: p_1 < p_2$$

(c) We have

$$\hat{p}_1 = \frac{1}{7897} = 0.000123,$$

$$\hat{p}_2 = \frac{91}{7899} = 0.011520$$

The pooled proportion

$$\hat{p} = \frac{1 + 91}{7897 + 7899} = 0.00582.$$

The standard error is

$$SE = \sqrt{0.00582(1 - 0.00582) \left( \frac{1}{7897} + \frac{1}{7899} \right)} = 0.00121.$$

Dividing the difference in proportions by the standard error, we have

$$z = \frac{0.000123 - 0.011520}{\sqrt{0.00582(1 - 0.00582) \left( \frac{1}{7897} + \frac{1}{7899} \right)}} = -9.4.$$

(d) Since the z-score is off the charts, the p-value is approximately zero.

(e) The study provides strong evidence that the vaccine is effective. The p-value tells us that the chance of the difference in HPV infection rates happening by chance is essentially zero, so we conclude that the vaccine is effective.

(f) The randomization (which women got the vaccine and which got the placebo was decided randomly) and the blinding (the women who got the placebo will not have known that this was not the vaccine) means that confounding variables and the Hawthorne effect were eliminated.