

Math 263
Quiz 2

Name _____

Benford Law's describes the distribution of first digits in the numbers in an honest IRS tax return. (Fraudulent tax returns are sometimes identified by the fact that the numbers do not have this distribution.)

Let X be the first digit of a number whose first digit is distributed according to Benford's law. Part of the pdf and cdf are given in the table.

First digit	1	2	3	4	5	6	7	8	9
pdf	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
cdf	0.301	0.477	0.602	0.699	0.778	0.845	0.903	0.954	1

(a) Find $P(X = 9)$

$$0.046$$

(b) Find $P(X \leq 2)$

$$0.301 + 0.176 = 0.477$$

(c) Fill in the ?? in the bottom row of the table.

$$P(X \leq 7) = 0.954 - 0.051 = 0.903.$$

(Or you could add the pdf for 1,2,3, etc. But this is slower!)

(d) Mark each of the following statements T (true) or F(false). (The expected value is the mean.) No reasons needed.

F The expected value of X is one of the integers from 1 to 9 (inclusive)

T The expected value of X is less than 5

F The expected value of X is more than 5

F The standard deviation of X is less than 1

T The standard deviation of X is more than 1

(e) Write the first two terms of the sum used to calculate the expected value using the values for this random variable.

$$E(X) = \mu = 0.301 \cdot 1 + 0.176 \cdot 2 + \dots$$

(f) Write the first two terms of the sum used to calculate the variance using the values for this random variable. (You can write μ instead of a number for the expected value.)

If μ is the expected value

$$V(X) = \sigma^2 = 0.301 \cdot (1 - \mu)^2 + 0.176 \cdot (2 - \mu)^2 + \dots$$