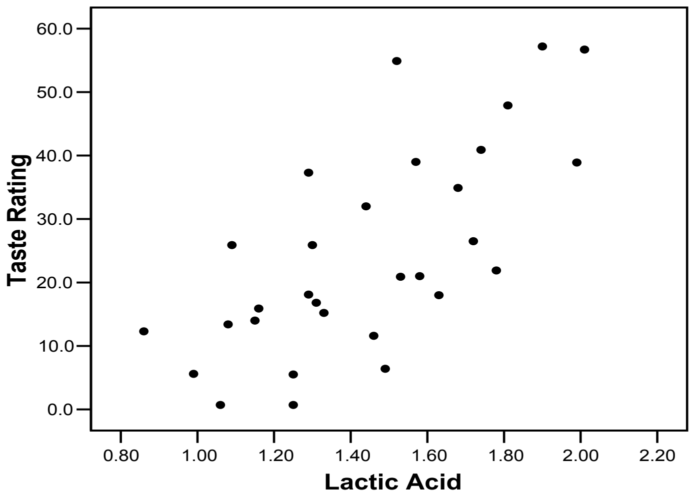
Mathematics 263 Section 6 Final Examination Solutions Spring 2013

1. As Swiss cheese matures, a variety of chemical processes take place. In a study, samples of cheese were analyzed for lactic acid concentration and were subjected to taste tests. A scatterplot of the observed data is shown below:



What is a plausible value for the correlation, r, between lactic acid concentration and taste rating?

(a) 0.999
(b) 0.7
(c) 0.07
(d) -0.07
(e) -0.7

2. In the last mayoral election in a city, 47% of the adults over the age of 65 voted Republican. A researcher wants to decide if the proportion of adults over 65 in the city who plan to vote Republican in the next mayoral election has changed. Let p represent the proportion of all adults over 65 in the city who plan to vote Republican in the next mayoral election. Which of the following null and alternate hypotheses should the researcher use?

(a) $H_0: p = 0.47$ versus $H_a: p < 0.47$
(b) $H_0: p = 0.47$ versus $H_a: p \neq 0.47$
(c) $H_0: p = 0.47$ versus $H_a: p > 0.47$
(d) $H_0: p = 0.5$ versus $H_a: p < 0.5$
(e) $H_0: p = 0.5$ versus $H_a: p \neq 0.5$
(f) $H_0: p = 0.5$ versus $H_a: p > 0.5$

3. A study collected data on two quantitative variables, x and y. A scatterplot of this data showed a straight-line relationship and the correlation was – 0.92. This tells us that

(a) All of the data values for the two variables lie on a straight line.
(b) There is little reason to believe that the two variables are linearly related.
(c) There is a strong linear relationship between the two variables with larger values of x tending to be associated with larger values of y.
(d) There is a strong linear relationship between x and y with smaller x values tending to be associated with larger values of y.
(e) There is a weak linear relationship between x and y with smaller x values tending to be associated with smaller values of y.

4. The length of gestation until birth for pregnant women is a random variable that is normally distributed with a mean $\mu = 282$ days and standard deviation $\sigma = 11$ days. Births with gestation times of 258 days or less are considered to be premature. What is the probability that a randomly selected pregnant woman will give birth to a premature baby?

(a) 0.2182
(b) 0.0146
(c) 0.0018
(d) 0.0183
(e) Not within ± 0.001 of any of the above.

5. Scores on a standardized test are normally distributed, with mean 35 and standard deviation 14. Only 5% of the test takers scored higher than Alejandra. What was her score?

(a) 7
(b) 49
(c) 58
(d) 63
(e) 77

6. Suppose exactly 50% of a population supports a candidate, what is the probability that a sample of 450 shows 52% support or more?

(a) 0.02
(b) 0.198
(c) 0.48
(d) 0.52
(e) 0.802

7. (Continuation of Problem # 6) Suppose the sample size for this poll was increased, but all the other quantities remained constant. How would the probability calculated in Problem #6 change?

(a) The new probability would be larger than before.
(b) The new probability would be smaller than before.
(c) The new probability would be the same as before.
(d) Need further information to tell.

8. An experiment was conducted to see if a new drug changed the mean time to recovery in comparison to the standard drug. The mean time to recovery for the standard drug is 26 days. Following a randomized experiment involving 65 patients, a 95% confidence interval was constructed for the mean recovery time (in days) for patients on the new drug; it was $(24.6, 27.8)$. Based on this interval, what can you conclude concerning the new drug relative to the standard drug?

(a) At the 0.05 significance level, there is evidence that the new drug is better than the standard drug.
(b) The experimenter should reject the claim that the new drug is the same as the standard drug with respect to mean recovery time.
(c) There is insufficient evidence to reject the claim that there is no difference between the new drug and the standard drug with respect to mean recovery time.
(d) The confidence interval contains the hypothesized value for μ and hence it is significant at the 0.05 level.
(e) There is reason to believe that the standard drug is better than the new drug and hence the new drug should not be prescribed.

Questions 9 – 15 use the following information:
In a study of test scores of entering college freshmen, a random sample of colleges across the nation is selected and the average SAT Math score for each of their freshman classes is recorded. The colleges are categorized as Public, Private, or Church. We want to know whether the freshmen entering the three types of colleges do equally well on the SAT Math. ANOVA computer output is shown below:

Source	Sum of squares	DF	Mean square	F	P-value
Between Groups	63906.2	2	31953.1	5.696	0.005
Within Groups (Error)	353440.2	63	5610.2		
Total	417346.4	65			

9. How many colleges were included in the study?

(a) 3
(b) 63
(c) 64
(d) 65
(e) 66

10. What is the value of the Mean Squares for Between Groups?

(a) 5,610.2
(b) 31,953.1
(c) 63,906.2
(d) 127,812.4
(e) 353,440.2

11. Under the null hypothesis of equal population means in the different types of colleges, what is the distribution of the test statistic? [Note: $F(a,b)$ is the F-distribution with $df = a, b$, and $N(a,b)$ is the normal distribution with $\mu = a$ and $\sigma = b$, and $T(k)$ is the T-distribution with $df = k$.]

(a) $F(2,63)$
(b) $F(2,65)$
(c) $N(0,3)$
(d) $N(2,63)$
(e) $T(63)$

12. The value of the F-statistic in the ANOVA table is 5.696 and the p-value is 0.005. If we draw the F-distribution and mark the value of 5.696 on the x-axis, how do we indicate the P-value on the graph?

(a) The area under the curve to the left of 5.696.
(b) The area under the curve to the right of 5.696.
(c) The area under the curve between -5.696 and 5.696.
(d) The area under the curve to left of -5.696 together with the area to the right of 5.696.
(e) Twice the area under the curve to the right of 5.696.

13. At a significance level of 0.05, what is the conclusion about the average SAT Math scores?

(a) The average SAT Math scores for freshmen attending colleges with the three different affiliations appear to be the same.
(b) Each of the three average SAT Math scores for freshmen attending colleges with the three different affiliations appear to be different.
(c) It appears that freshmen attending at least one of the three different types of college have a different average SAT Math score.
(d) Freshmen at one type of affiliated college have a significantly better average SAT Math score than the other two.
(e) Two of the types of institutions have the same average SAT scores; the third is different.

14. Which of the following statements about the SAT Math ANOVA table is (are) true?

(a) Sums of squares represent variation present in the data and they are calculated by summing squared deviations.
(b) There are three distinct sources of variation represented in the table: between groups, within groups (error), and total
(c) The sum of squares for total (SST) is composed of two parts, one due to groups (SSG) and one due to error (SSE).
(d) Mean squares are calculated by dividing the corresponding sum of squares by its degrees of freedom.
(e) All of the above are true.

15. (**3 points**) One of the assumptions in ANOVA is that the population standard deviations are equal. Mark each of the following statement as True (T) or False (F).

___T__We could use side-by-side boxplots to assess if this assumption of equal population standard deviations seems reasonable.

___F__As long as the ratio of the largest to the smallest sample standard deviation is greater than 2, then the assumption seems to be satisfied.

___T__In the SAT study, an estimate for the common standard deviation s in the three populations equals 74.90.

16. **(4 points**) Which of the following are correct interpretations of the p-value? Mark each one True (**T**) or False (**F**)

___F_The probability that the null hypothesis is false

__F__The probability that the alternate hypothesis is true

__T__The probability that, if the null hypothesis is true, we obtain by chance sample data is as extreme as we got or more so.

___F_ The probability that, if the alternate hypothesis is true, we obtain by chance sample data is as extreme as we got or more so.

17. This is a one-sample test of proportions. The baseline rate is given out of 22,000, but it is a population parameter. (The second 22,000 is not a sample.) The population is all men; the sample is the 22,000 who had vasectomies.

The population prevalence rate is $p = 70/22{,}000 = 0.00318 = 0.318\%$.

(a) The sample prevalence rate is $\hat{p} = 113/22{,}000 = 0.00514 = 0.514\%$.
   (i) Null hypothesis: $H_o\colon p = 0.00318$
   (ii) Alternate hypothesis: $H_a\colon p \neq 0.00318$
   (iii) Then $\hat{p} = 113/22{,}000 = 0.00514 = 0.514\%$, so

$$z = \frac{0.00514 - 0.00318}{\sqrt{\frac{0.00318(1-0.00318)}{22{,}000}}} = 5.16$$

   (iv) This is a large z-value, so the p-value is small. (About 10^{-7}.)
   (v) We reject the null hypothesis and accept the alternate hypothesis. We conclude that cancer rates are higher in men who have had vasectomies.

(b) No, because this is an observational study. The men who had vasectomies are not a random sample. There may be confounding variables.

18. This is a regression study with response variable $y = \text{Thickness (in mm)}$ and explanatory variable $x = \text{PCB concentration (in ppm)}$.

(a) Reading constants from the table, we have Thickness $= 0.3639 - 0.0002 \cdot \text{PCB}$.
(b) If PCB $= 300$, then predicted Thickness $= 0.3639 - 0.0002 \cdot 300 = 0.3039$ mm.
(c) The units are millimeters
(d) The model predicts that if there were no PCB, the shell thickness would be 0.3639 mm.
(e) The units are millimeters per ppm.
(f) The model predicts that for each additional ppm of PCB, the thickness of the shell is reduced, on average, by 0.0002 mm.
(g) Two-sided test of slope.

(i) Slope is 0
(ii) Slope not equal to 0
(iii) $t = 1.4350$
(iv) $p = 0.1567$
(v) We cannot reject the null hypothesis. We do not have sufficient evidence to say that the slope is nonzero.

(h) No, we cannot say that there is relationship between PCB concentration and shell thickness.

19. The variable is categorical: each person opposes the mine or not.

(a) The 95% margin of error is given by

$$ME = 1.96\sqrt{\frac{0.5(1-0.5)}{625}} = 0.0392 = 3.92\%.$$

(b) The proportion of the sample opposing the mine is $\hat{p} = 287/625 = 0.4592$. Thus the 95% confidence interval is given by

$$(0.4592 - 0.0392, 0.4592 + 0.0392) = (0.4200, 0.4984) = (42\%, 49.84\%).$$

(c) Mark each of the following statements about the confidence interval as True (T) or False (F):
__F__ 95% of the sample means of all samples of size 625 lie in this interval; 5% do not
__F__There's a 95% chance that the number opposing the mine in a sample of 625 lies in this interval
__T__95% of the intervals generated by this method contain the population proportion; 5% do not.
__F__95% of the population proportions lie in this interval; 5% do not.
__T__There's a 95% chance that the population proportion lies in this interval.

(d) The new proportion is $\hat{p} = 344/625 = 0.5504$. The 95% margin of error remains the same, so the new confidence interval is

$$(0.5504 - 0.0392, 0.5504 + 0.0392) = (0.5112, 0.5896) = (51.12\%, 58.96\%).$$

(e) Yes, because even the lower end of the interval, 51.12%, is above 50%.

20. The variable is quantitative, with population mean 130 mm and standard deviation 20 mm.

(a) For a single random sample of 200 people
  (i) Mean is approximately 130 mm
  (ii) Standard deviation is approximately 20 mm

(b) For the three sampling distributions:
  (i) Means:

Mean for $n = 20$ is ___130_____

Mean for $n = 50$ is ___130_____

Mean for $n = 200$ is ___130_____

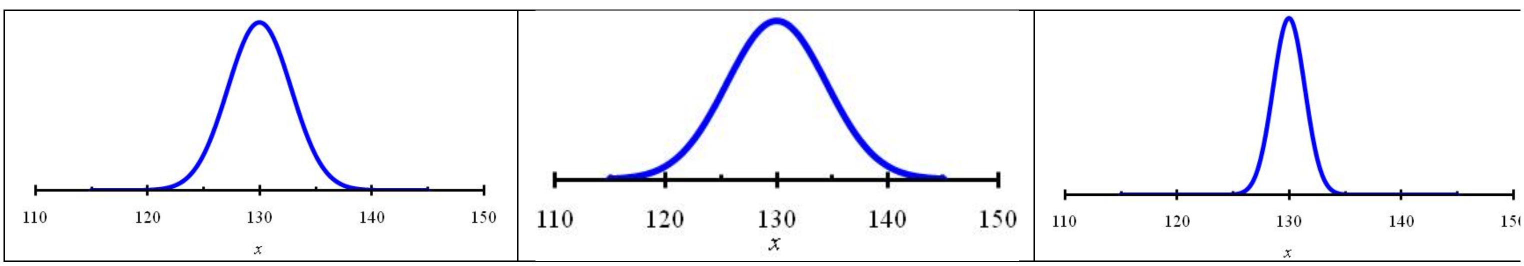
  (ii) Which of the three distributions has the smallest standard deviation? (*Check one*)

SD is smallest for $n = 20$______ or $n = 50$_____ or $n = 200$___X___

  (iii) For $n = 200$, we have

$$SD = \frac{\sigma}{\sqrt{n}} = \frac{20}{\sqrt{200}} = 1.414 \text{ mm.}$$

  (iv) Match the three systolic blood pressure sampling distributions with their graphs:



Sample size: ___50_____ Sample size: ___20_____ Sample size: ____200____

21. The events in the table are non-overlapping (so mutually exclusive.).

(a) Probability of 8 or more good years = $0.13 + 0.05 + 0.01 = 0.19$.
(b) Probability of at least one bad year $= 1 -$ Probability of 10 good years $= 1 - 0.01 = 0.99$
(c) The conditional probability is larger because once there have been 3 good years, some of the events that preclude 8 good years are already known not to have happened.
(d) We have

$$P(\text{8 or more good years}|\text{4 or more good years}) = \frac{(P(\text{8 or more good years } \mathit{and} \text{ 4 or more good years})}{P(\text{4 or more good years})}$$

$$= \frac{(P(\text{8 or more good years})}{P(\text{4 or more good years})} = \frac{0.19}{1 - 0.13} = 0.218.$$