

Math 263 Section 005: Class 2 : Normal Distribution and z-scores

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Course Policies and Information at <http://math.arizona.edu/~dhh/263-14.html>. You need a WebAssign account for the course. Go to www.webassign.net. The class key is **arizona 7989 8719**

Class 2 covers the normal distribution, including standard normal z-values, in Section 1.3 in Introduction to the Practice of Statistics, 7-th edn, by D. Moore, G. McCabe, B. Craig. (W.H. Freeman, 2012)

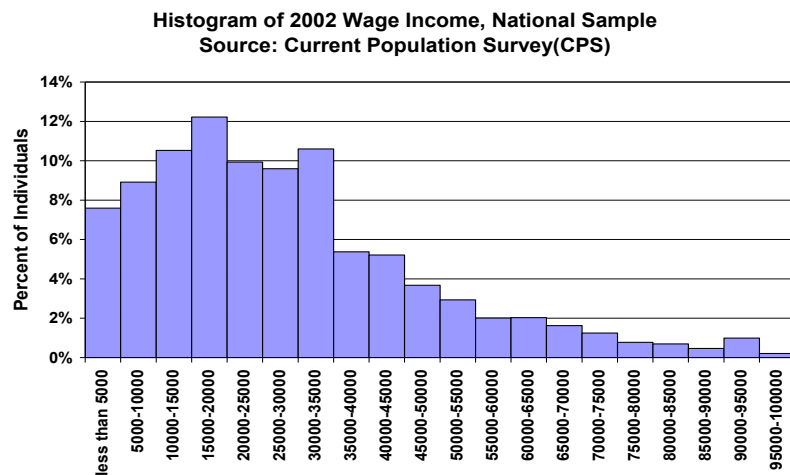
TYPES of VARIABLE

- **Discrete variables:** Age in years, number of days temperature goes over 100°F, class size
- **Continuous variables:** Exact temperature in °F, sea level, atmospheric pressure

What is a distribution? What does it tell you?

Histogram: Picturing the Distribution of Discrete Variable

What are the axes when a data set is represented graphically using a histogram? What would a distribution of income in a city look like?

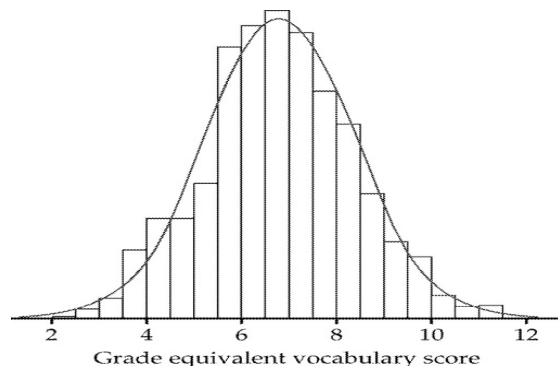


- **Horizontal axis:**
- **Vertical axis:**

- **Shape:**

Density Curve: Picturing the Distribution of a Continuous Variable

The histogram shows the actual vocabulary scores of a group of 7-th grade children. The smooth curve is the idealized curve what we imagine we would get if we took the population of all such children and made the bin widths very small. The smooth curve is called a *density curve*.



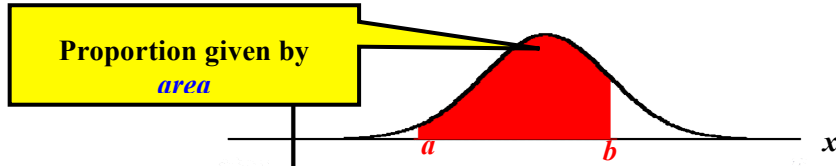
What do you notice about the smooth curve?

Using a distribution to measure proportions and probabilities

For a discrete variable, such as age in years, we can calculate the proportion of the population that has a specific value—for example, the proportion of 10-year olds in a school.

For a continuous variable, such as the quantity of oil discovered in a region. We expect no discoveries to lead to *exactly* 2 trillion cubic feet of oil, for example, so the proportion having any particular value is zero. We can only calculate the proportion *between* two values. Thus we have:

- **Histogram:** Proportion, or probability, is height of bars
- **Density Curve:** Proportion, or probability, is area under curve

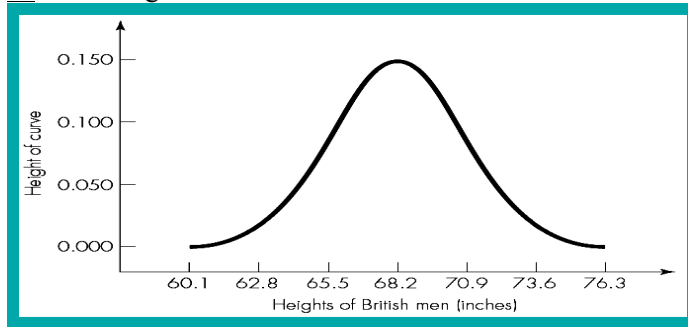


The proportion of data values lying between two values is the *area* under the density curve between these values.

The NORMAL DISTRIBUTION

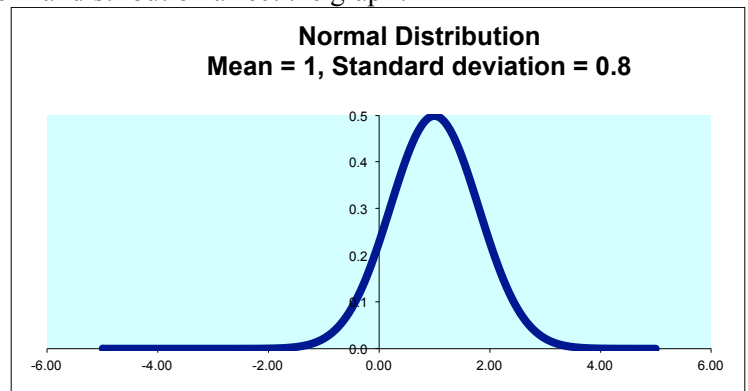
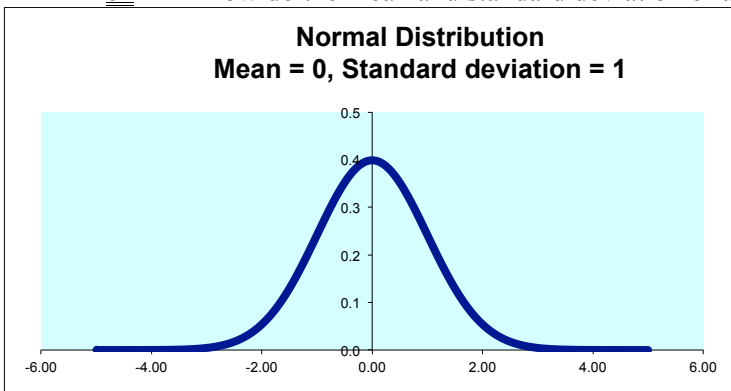
Represented by a bell shaped curve, bunched around the mean.

ex Heights of men¹. Estimate the mean and standard deviation.



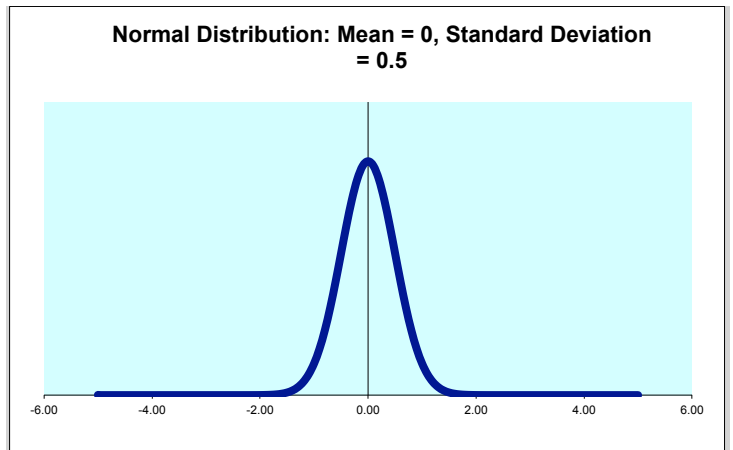
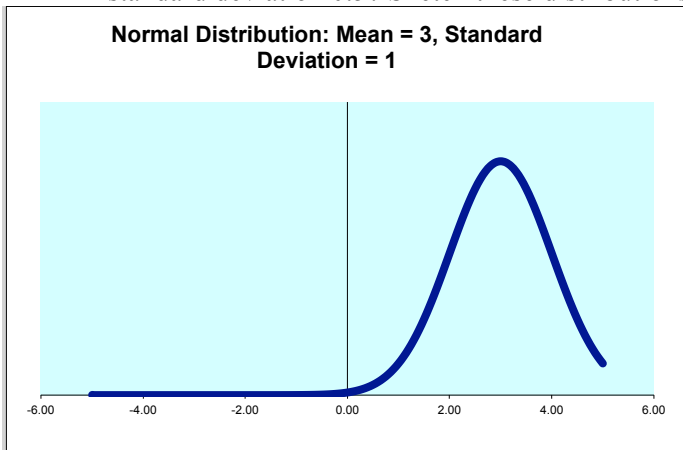
Mean: 68.2 inches
Standard Deviation: 2.7 inches

ex How do the mean and standard deviation of a normal distribution affect the graph?



¹ From Jessica Utts, *Seeing Through Statistics* 3rd edition. Thomson-Brooks Cole.

ex: How does a normal distribution look with mean 3 and standard deviation 1? With mean 0 and standard deviation 0.5? Sketch these distributions.



Ex. Which of the following quantities are likely to be normally distributed? Make rough sketch.

- Incomes in a city

Scores on a standardized test (for example ACT, SAT)

“Rule of Thumb”: for the Normal Distribution

For all normal curves, it can be shown that, approximately

- 68% of the data lies within 1 std dev of mean
- 95% of the data lies within 2 std dev of mean
- 99.7% of the data lies within 3 std dev of mean

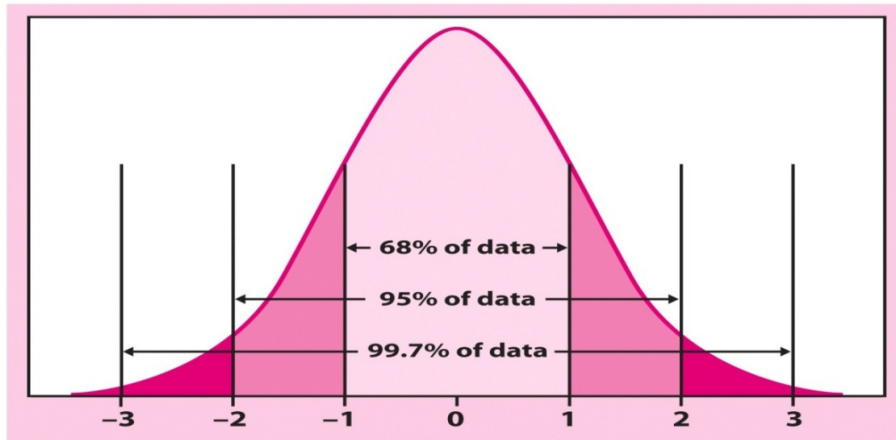


Figure 13-8
Statistics: Concepts and Controversies, Sixth Edition
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ex: Women’s heights are normally distributed with mean 65 inches (165 cm) and standard deviation 2.5 inches (6.4 cm).

- (a) What proportion of women are less than 60 inches? (5ft)
- (b) What proportion of women are less than 70 inches?
- (c) What proportion is more than 72 inches? (6ft)

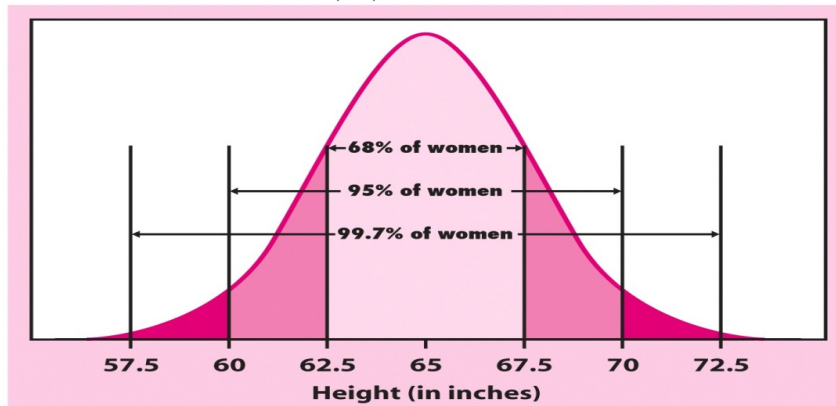


Figure 13-9
Statistics: Concepts and Controversies, Sixth Edition
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How do we compute proportions if the value given is not an exact multiple of standard deviations away from the mean? (Like the 72)? We use the z-score (or z-value), defined next.

THE Z-SCORE: Comparing Values in Different Normal Distributions

To compare values from different normal distributions (with different means and standard deviations), we find how many standard deviations each value is above or below its mean. This is the **z-score**:

$$z = \frac{\text{Value} - \text{Mean}}{\text{Standard deviation}}$$

The values of z have the **standard normal distribution**, with mean = 0 and standard deviation = 1.

Ex: On the 2008 SAT, which of the following scores represents the best performance:² 580 on reading, 595 on math, or 575 on writing?

| | Reading | Math | Writing |
|---------------------------|----------------|-------------|----------------|
| Mean | 501 | 515 | 493 |
| Standard Deviation | 112 | 116 | 111 |

Z-TABLES

The normal table shows the z -values to the left (with the second decimal place across the top); the body of the table shows the proportion of values to the left of each z -value. (See picture at top of table.)

Finding the Proportion of Data in a Normal Distribution using the Table

ex Find the proportion of data that has z -score less than 0.7.

ex Find the proportion with z -score above 1.2.

ex Find the proportion with z -score between -0.20 and 1.4 .

ex What z -score is at the 70th percentile?

ex What z -score has 75% of the data above it?

ex: Find the proportion of women who are shorter than 72 inches. (Heights are normally distributed with mean 65 inches and standard deviation 2.5 inches.)

² The College Board's *Total Group Profile Report: 2009 College-Bound Seniors*

Ex: Find the proportion of women taller than 72 inches.

Ex: How tall is a woman who is at the 30th percentile in height?

Ex: What percentile is a SAT reading score of 700? (Mean reading score is 503; standard deviation 113).

ex What SAT math score is at the 90th percentile? (Mean math score is 518; standard deviation 115.)

Finding proportions using a TI-83/84

To find proportion between a and b , look under “Distr” menu and use
normalcdf (a, b , mean, standard deviation)

To find the x value that corresponds to a proportion p , use
invNorm(p , mean, standard deviation)

Finding proportions using Excel

To find the proportion of the data to the left of x , use
= NORMDIST (x , mean, standard deviation, true).

To find the x value that corresponds to a proportion p , use
= NORMINV (p , mean, standard deviation)