Class 22: Analysis of Variance (Text: Sections 12.1)

The F-test: Tests whether the means of two or more populations are equal.
Now we do examples with more than two samples, which cannot be done by the *T*-test.

> **Null hypothesis:** The means of all the populations are equal.
> **Alternative hypothesis:** The means of all the populations are not all equal. Thus at least one population has a different mean.

Assumptions Underlying the F-Test

For the F-statistic to have the F- distribution, we have to assume

- Each sample comes from a normal population
- The population standard deviations are all equal

The means of the normal distributions do *not* have to all be equal—that's what we want to decide.

Fortunately, the F-Test is not very sensitive to unequal standard deviations,

- We can use the F-Test if the largest sample standard deviation is less than twice the smallest one.
- In other words, the largest variance should be no more than 4 times the smallest one

Pooled Variance

Let s_p^2 be the variance of all the samples pooled together. The estimate for this pooled variance is

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2 + \cdots + (n_k-1)s_k^2}{(n_1-1)+(n_2-1)+\cdots+(n_k-1)}.$$

The common population standard deviation is $\sqrt{s_p^2}$.

Relationship Between s_p^2 and SSE and MSE

For each sample, we have

$$s_i^2 = \frac{1}{n_i-1}\sum_{Group\ i}(x_{ij}-\bar{x}_i)^2 \qquad \text{so} \qquad (n_i-1)s_i^2 = \sum_{Group\ i}(x_{ij}-\bar{x}_i)^2.$$

Thus

$$SSE = \sum_{Groups}\sum_{j}(x_{ij}-\bar{x}_i)^2 = \sum_{i}(n_i-1)s_i^2.$$

Now $(n_1-1)+(n_2-1)+\cdots+(n_k-1) = N-k =$ DFE, so we have

$$MSE = \frac{SSE}{N-k} = s_p^2$$

Example: **The "fog index" measures the difficulty in a passage or writing.[1] A sample of passages from three magazines gave the following results.**

Scientific American	15.75	11.55	11.16	9.92	9.23	8.20
Newsweek	10.21	9.66	7.67	5.12	4.88	3.12
Sports Illustrated	9.17	8.44	6.10	5.78	5.58	5.36

Ex: Complete the table below:

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Scientific American	6	65.81	10.96833	7.004777
Newsweek	6	40.66	6.776667	8.122507
Sports Illustrated	6	40.43	6.738333	2.675217

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	70.92888	2	35.46444	5.976313	0.012336	3.68232
Within Groups	89.0125	15	5.934167			
Total	159.9414	17				

Ex: Does the data meet the conditions for the F-Test?

Ex: Is there a significant difference in fog indices between these magazines?

Ex: How do we tell which magazine(s) have different fog indices?

[1] From Shruptine and McVicar, reported by Wilde and Seber in *Chance Encounters* .The fog index is defined by Fog Index $= 0.4 \cdot$ Average #words per sentence + %words with $>$ 3syllables.

Ex: Does the type of cooking pot affect the iron content of food?[2]

Iron deficiency leads to anemia. In developing countries, iron has traditionally got into the food from iron cooking pots. But as heavy iron pots are replaced by lighter, cheaper aluminum pots, there is a concern that anemia and malnutrition may result. Use the data to decide if there is a significant relationship between type of pot and iron content in food.

	Iron Content (mg per 100 gm of food)			
Aluminum	1.77	2.36	1.96	2.14
Clay	2.27	1.28	2.48	2.68
Iron	5.27	5.17	4.06	4.22

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Aluminum	4	8.23	2.0575	0.063492
Clay	4	8.71	2.1775	0.386025
Iron	4	18.72	4.68	0.394733

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	17.53922	2	8.769608	31.16236	9.01E-05	4.256495
Within Groups	2.53275	9	0.281417			
Total	20.07197	11				

Ex: What do the summary statistics suggest?

Ex: Are the conditions for the F-Test met?

Ex: Perform the hypothesis test. What can you conclude?

[2] Text, Problem 12.49, reported from A. A. Adish, "Effect of consumption of food cooked in iron pots on iron status and growth of young children: a randomized trial", *The Lancet* (1999).

Ex: If we combine clay and aluminum, does the food cooked in iron pots have significantly higher iron content?

Iron	5.27	5.17	4.06	4.22				
Aluminum and Clay	1.77	2.36	1.96	2.14	2.27	1.28	2.48	2.68

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Iron	4	18.72	4.68	0.394733
Aluminum and Clay	8	16.94	2.1175	0.196764

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	17.51042	1	17.51042	68.35868	8.81E-06	4.964603
Within Groups	2.56155	10	0.256155			
Total	20.07197	11				

Ex: Does bread loose vitamins when it is stored?[3]

	Days 0	Days 1	Days 3	Days 5	Days 7
Vitamin C	47.62	40.45	21.25	13.18	8.51
	49.79	43.46	22.34	11.65	8.13

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Days 0	2	97.41	48.705	2.35445
Days 1	2	83.91	41.955	4.53005
Days 3	2	43.59	21.795	0.59405
Days 5	2	24.83	12.415	1.17045
Days 7	2	16.64	8.32	0.0722

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	2565.721	4	641.4302	367.742	2.33E-06	5.192168
Within Groups	8.7	5	1.74			
Total	2574.442	9				

[3] Text, Problem 12.29, reported from H. Park et al "Fortifying bread with each of three antioxidants, *Cereal Chemistry* (1997).