To splash or not to splash . . .

The effect of Weber number and spread factor of a water droplet impinging on a super-hydrophobic substrate

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Outline

Motivation / Applications of Low Viscous Impinging

http://www.cardensdesign.com/photography/CD00067.jpg

- Agriculture
- Inkjet printers
- Windshields
- Clothing

Experiment

Photo credits to Robert Reinking

- Water droplet released from rest

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- Droplet impinged upon super-hydrophobic substrate

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- Droplet spreads/splashes, and subsequently rebounds

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- Water droplet released from rest
- Droplet impinged upon super-hydrophobic substrate
- Droplet spreads/splashes, and subsequently rebounds
- Look for predictors of droplet impact behavior

Background and Previous Work

- Foote - Numerical integration of Navier-Stokes equations
- Sikalo - Experimental results of flat and inclined surface
- Attane - Theoretical model of energy balance equation including dissipation

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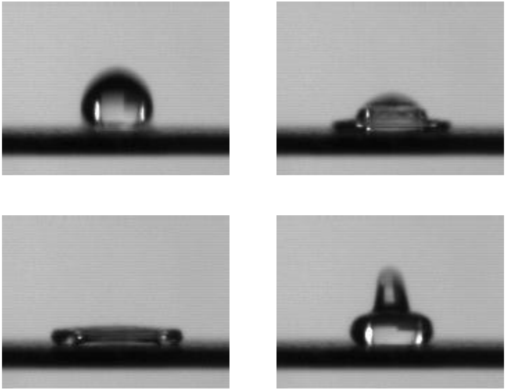
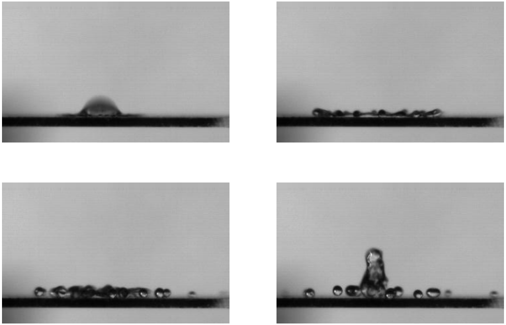
- Durickovic and Varland - Performed the experiments from which we collected our data
- Smith and Gordon - Damped harmonic oscillator approximation

Spread Sequence



The droplet flattens to a maximum diameter, then rebounds in the form of a Worthington jet.

Splash Sequence



The droplet spreads upon impact and finger formations appear. Kinetic energy overcomes surface tension, and the droplet splashes as several smaller pieces break off prior to rebound.

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- Contact Angle (θ_c) - Interior angle from substrate to surface of droplet

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- Ohnesorge Number (Oh) - Ratio of viscous forces to surface tension

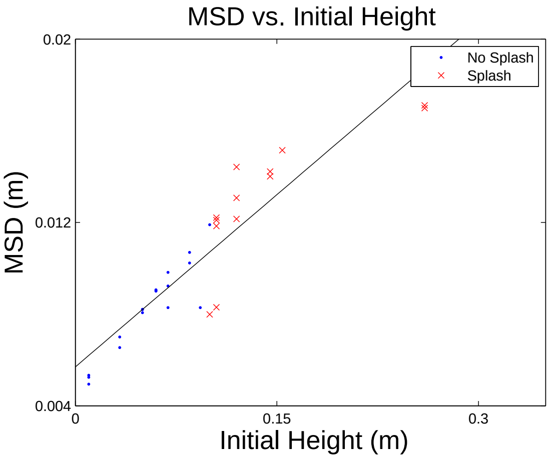
$$Oh = \frac{\sigma}{\sqrt{\gamma \rho D}} = \sqrt{We}/Re$$

Outline

Data Ranges

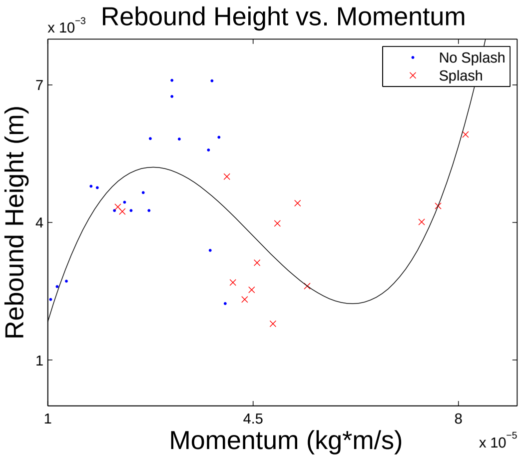
Initial Height (cm)	Number of Video Samples	Splash on Impact	We Range	β Range
1	3	0	9.6 – 10.4	1.39 – 1.42
3.3	2	0	30.7 – 31.4	1.89 – 1.99
5	3	0	47.8 – 49.9	2.22 – 2.28
6	3	0	61.5 – 62.3	2.34 – 2.38
6.9	3	0	66.1 – 73.4	2.33 – 2.59
8.5	3	0	87.6 – 88.7	2.68 – 2.79
9.3	1	0	78.1	2.66
10	2	1	83.7 – 102.5	2.57 – 3.13
10.5	4	4	88.2 – 110.6	2.66 – 3.20
12	3	3	119.6 – 127.5	3.08 – 3.90
14.5	2	2	152.7 – 154.3	3.59 – 3.60
15.4	1	1	156.2	4.03
26	3	3	278.0 – 287.1	4.18 – 4.56

Maximum Spreading Diameter vs. Initial Height



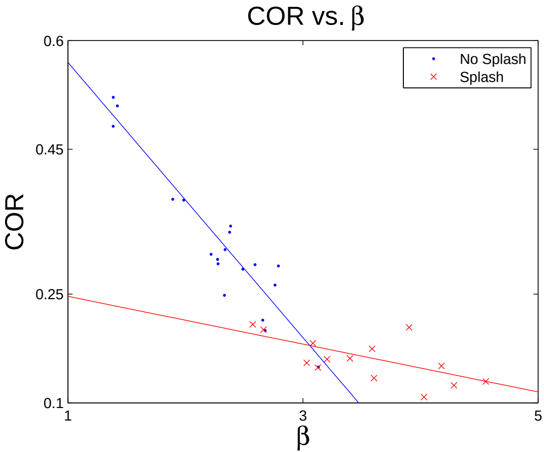
Linear fit $y = 0.05011x + 0.005696$

Rebound Height vs. Momentum



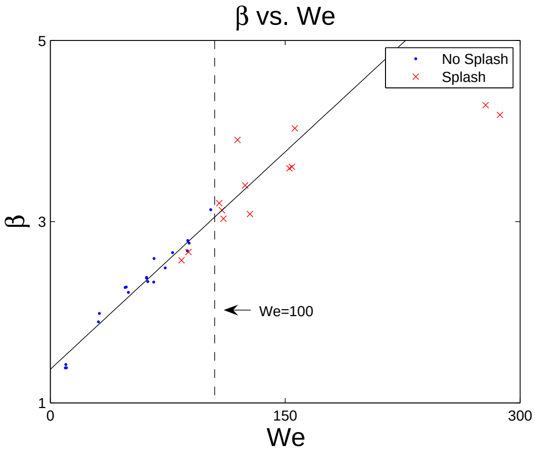
Cubic fit $y = 1.52e11x^3 - 2.05e7x^2 + 790x - .0042$

Coefficient of Restitution vs. Spread Factor



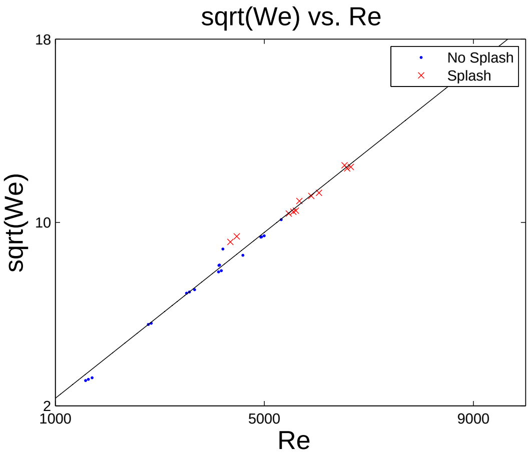
Linear fit (*blue*) $y = -0.19x + 0.76$, Linear fit (*red*) $y = -0.03x + 0.28$

Spread Factor vs. Weber Number



Linear fit $y = 0.016x + 1.37$

$\sqrt{\text{Weber Number}}$ vs. Reynolds Number



Linear fit $y = 0.0018x + 0.515$

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Theoretical Model Assumptions

Time-Dependent Model

$$\frac{d}{dt}\left(E_k + E_g + E_\gamma\right) + \Delta = 0$$

- 2-dimensional motion of a water droplet

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- Droplet released from rest
- Maintains azimuthal symmetry
- Rotational kinetic energy is negligible

Conservative Energy Factors

Time-Dependent Model

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$$E_k = \frac{\pi}{12}\rho D_0^3 v^2$$

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- Surface Tension

$$E_\gamma = \pi\gamma[D_0^2 - R^2 \cos\theta_c]$$

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3. Rebound
   - Aerodynamic drag force
   - Viscous flow within the droplet

Dissipation During Initial Decent

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$$\frac{d}{dt}\left(E_k + E_g + E_\gamma\right) - F_d \cdot v = 0$$

- Dissipation dominated by drag force
- For *Re* between 3×10^2 - 3×10^5, the drag force acting on a smooth sphere $\propto v^2$.

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$$\Delta = -F_d \cdot v$$

$$F_d = \frac{\pi}{8}\rho D_0^2 c_D v^2$$

Energy-Balance Model for Initial Descent

Time-Dependent Model

$$\frac{d}{dt}\left(E_k + E_g\right) - F_d \cdot v = 0$$

Recall $\frac{d}{dt}E_\gamma = 0$ due to constant surface area, we differentiate and solve for $\dot{v}$ . . .

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$$\dot{v} = -g\left(1 - \frac{3c_D}{4gD_0}v^2\right)$$

Non-Dimensionalized System

Time-Dependent Model

$$\frac{d}{dt}(E_k + E_g) - F_d \cdot v = 0$$

System of Differential Equatons

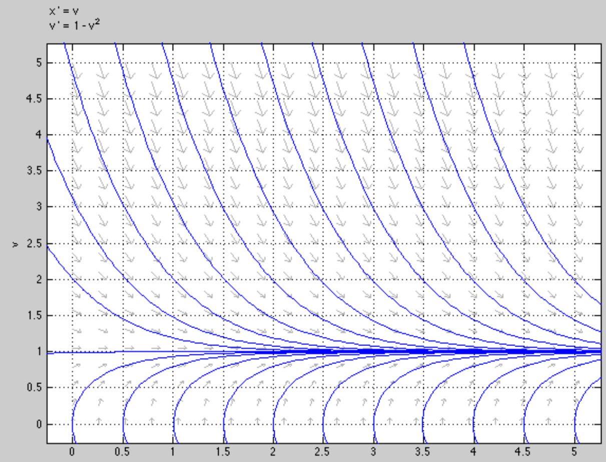
$$\begin{aligned} \mathcal{X}' &= \nu \\ \nu' &= 1 - \nu^2 \end{aligned}$$

Solution

$$\nu(\tau) = \tanh(\tau)$$

Non-Dimensionalized Phase Portrait

Velocity vs. Position



Energy-Balance models for Impact & Rebound

Time-Dependent Model

$$\frac{d}{dt}\left(E_k + E_g + E_\gamma\right) + \Delta = 0$$

- During impact, we no longer have a constant E_γ, as the wetting radius is changing. We also must include viscous flow within the droplet, which was assumed negligible for initial drop.

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- Rebound stage incorporates both aerodynamic and viscous dissipation, as the droplet experiences significant surface vibrations.
- Coupled with the loss of mass and energy during splashing, solving these systems is currently beyond the scope of this project

Outline

Future Research

- Inclusion of Navier-Stokes equations to understand dissipation effects at later stages.
- Analyze droplet behavior on different substrates (*hydrophobic/hydrophilic*).
- Alternate camera angle to analyze spreading and symmetrical growth of finger formation.

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Conclusions

- Found critical *We* of 100 to predict splashing for water droplets.
- Experimental *Oh* of 1.8×10^{-3} confirms accepted *Oh* of $\approx 1.7 \times 10^{-3}$ for water droplet of $We = 30$. (*Sikalo*)
- Non-dimensionalized system to model droplet dynamics prior to impact.

References

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- Smith, K., Gordon, C. *Impact of Weber number on the behavior of an impinging water droplet* (2006).

Questions, Acknowledgements

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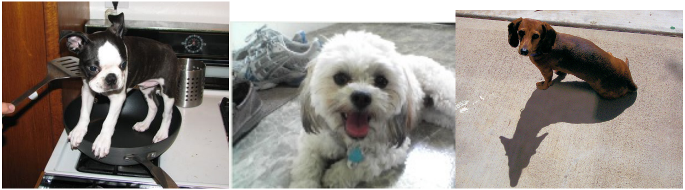

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