Unit 4: Sections 10.1 - 10.3 and 11.1 - 11.6

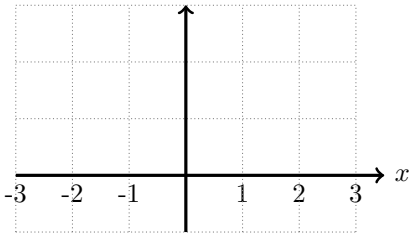
In chapter 10, we are going to use what we learned about sequences and series to approximate complicated functions with simple polynomials.

Set-up:

1. Go to the website
   `http://mathlets.org/mathlets/taylor-polynomials/`
2. Select the function $f(x) = 1/(1+x^2)$ from the drop-down list on the bottom left.
3. Move the slider on the bottom to $n = 2$
4. Check the box on the bottem right for "terms".

The x-axis is also a slider that allows you to select where to focus your approximation. Place the slider on the point $x = -0.5$, draw what you see on the graph below and fill in the terms of the expansion on the right.

Approximating Polynomial



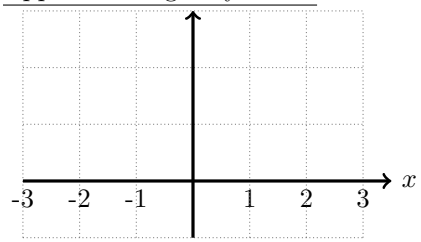
$p(x) \quad =$

$\quad\quad +$

The blue curve is the function $f(x) = \frac{1}{1+x^2}$ and the yellow line is the linear approximation to $f(x)$ at $x = -0.5$. Now press the play button and describe what happens:

Now move the slider to $n = 2$. Draw the approximating polynomial at $x = 1$ and write down the terms shown.

Approximating Polynomial



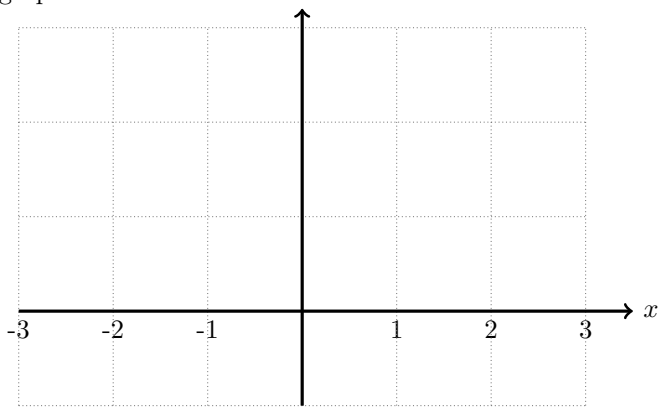
$p(x) \quad =$

$\quad\quad +$

$\quad\quad +$

Remember how a power series has a radius of convergence, which for some series is zero, for others finite, and for others it is infinte? Let's explore this idea a little further.

Select the function $f(x) = \arctan(x)$ and sketch it on the graph below.

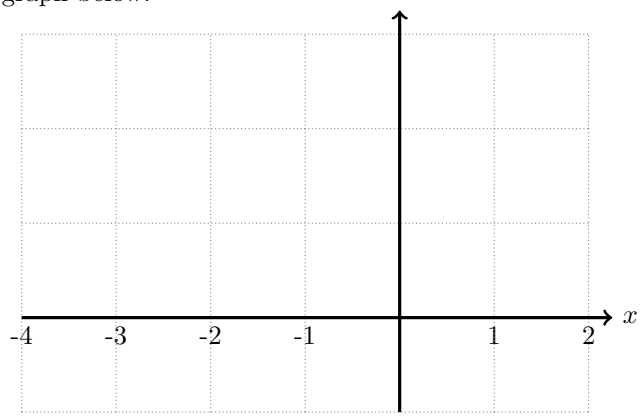


By moving the slider for $n = 1$, $n = 3$ and $n = 7$, sketch polynomial approximations to $f(x) = \arctan(x)$ 'focused' at $x = 0.5$.

Circle one: The radius of convergence appears to be (a) zero, (b) finite and non-zero, (c) infinte.

Circle one: The terms are getting smaller, bigger, neither.

Now select the function $f(x) = e^x$ and sketch it on the graph below.



By moving the slider for $n = 1$, $n = 3$ and $n = 7$, sketch polynomial approximations to $f(x) = e^x$ 'focused' at $x = 0$.

Circle one: The radius of convergence appears to be (a) zero, (b) finite and non-zero, (c) infinte.

Circle one: The terms are getting smaller, bigger, neither.

Taylor Polynomials

Section 10.1 & 10.2
November 8, 2016

Name:
Math 129 - 01

Understand	Use the terminology " Taylor Polynomial of degree n approximating $f(x)$ near $x = a$ ".
Understand	Know that a Taylor polynomial is a higher order generalization of tangent line approximations.
Understand	Know when a Taylor polynomial approximation will be exact.
Apply	Construct Taylor polynomials of a given degree approximating a function $f(x)$ near $x = a$ by taking derivatives of $f(x)$and evaluating them at $x = a$.

1. The coefficients of a Taylor Polynomial are found by taking derivatives of the function that is being approximated.

   For $f(x) = \dfrac{1}{1+x^2}$, calculate the following:

   $f'(x) =$

   $f'(1) \approx$

   $f''(x) =$

   $f''(1) \approx$

Repeat the calculations for $f(x) = e^x$

$f'(x) =$

$f'(0) \approx$

$f''(x) =$

$f''(0) \approx$

Quiz (Leave this space blank)