1) **[10 points]** True or false: Clearly indicate if the following statements are true or false.

a) _____ The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ is absolutely convergent.

b) _____ If $\lim_{n\to\infty} |a_n| = 0$, the series $\sum_{n=1}^{\infty} a_n$ converges.

c) _____ A sequence that is monotone and bounded must be convergent.

d) _____ $\sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+1} = 1 - \lim_{n\to\infty} \frac{1}{n+1} = 1$

e) _____ The work required to lift a two pound object five feet in the air is 10 J.

2) **[5 points]** Suppose the partial sums of a series are given by the equation

$$S_n = 17 + \frac{1}{\ln n}.$$

a) The series converges.

b) The series diverges by the integral test.

c) The series diverges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

d) The series diverges by ratio test.

3) **[5 points]** Consider the geometric series

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$$

. To what value does this series converge?

a) $-\frac{2}{3}$

b) 0

c) $\frac{2}{3}$

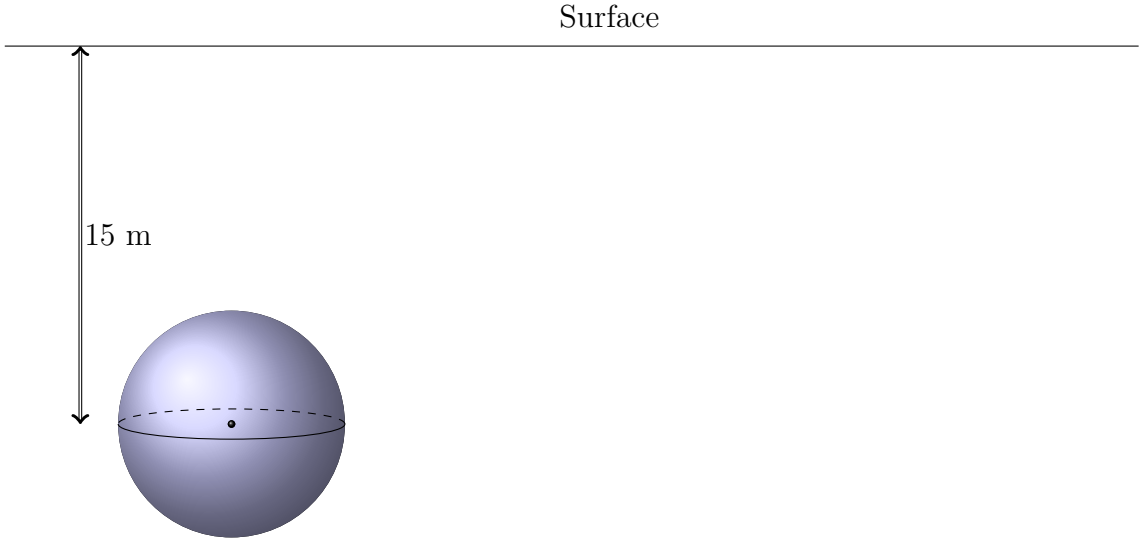
d) 3

e) 5

f) The series DIVERGES

4) **[25 points]** A spherical tank of radius 5 meters is buried in the ground. Suppose the center of the tank is 15 meters below the surface and the tank is completely full of water which has density 1000 kg/m^3.

a) Draw a slice (taken parallel to the ground) of water h meters from the center of the tank with a thickness Δh. Give an expression for the work required to pump the slice to the surface in terms of h and Δh. Use $g = 9.8$ m/s^2.



b) Use your result from part a to write an integral which gives the total work required to pump water from the tank to the surface. You do not need to solve this integral.

5) **[15 points]** Determine whether the series is convergent or divergent. Justify your answer by clearly stating which test you use and show that all requirements of the test are met.

$$\sum_{n=5}^{\infty} \frac{100n + n^{5/2}}{\sqrt{17n + n^8}}$$

6) **[15 points]** Determine whether the series is convergent or divergent. Justify your answer by clearly stating which test you use and show that all requirements of the test are met.

$$\sum_{n=0}^{\infty} \frac{(n+1)!(n+2)!}{3^n(2n+1)!}$$

7) **[25 points]** Consider the power series

$$\sum_{n=0}^{\infty} \frac{2^n (x-1)^n}{n!}.$$

a) Find the radius of convergence of this power series.

b) Using your result from part a, give the interval of convergence. You do not need to consider the endpoints of the interval if they exist.