1. Find the third-order Taylor polynomial of $(1 + x)\, e^x$ about the point $x = 0$. Compute the Taylor polynomial directly from the definition - do not use the table of known Taylor series. [*10 points*]
   Use the result above to calculate the exact value of the sum

$$\sum_{n=0}^{\infty}(-1)^n \frac{n+1}{n!}$$

   [*5 points*]

2. Padé approximants are rational functions used to approximate more complicated functions. In this problem, you will derive the Padé approximate to the exponential function. [*20 points*]
   (a) Let $f(x) = (1 + ax)/(1 + bx)$, where a and b are constants. Use any method you choose to write down the first four terms in the Taylor series for $f(x)$ about $x = 0$.
   (b) What is the radius of convergence of the Taylor Series in (a)?
   (c) Write down the first four terms in the Taylor series of $g(x) = e^x$ about $x = 0$.
   (d) Use your results from (a) and (c) to explain why selecting $a = 1/2$ and $b = -1/2$ makes $f(x)$ the best possible Padé approximate to $g(x)$ near $x = 0$.
   (e) Write **one** sentence explaining how you could provide a reasonable estimate on the difference, $f(x) - g(x)$, for values of x that are very close to 0.

3. Identify the following series as the series expansion of a particular function and use this information to find the sum of the series.

$$(\sqrt{e}-1) - \frac{(\sqrt{e}-1)^2}{2} + \frac{(\sqrt{e}-1)^3}{3} + \ldots$$

   [*10 points*]

4. For what value(s) of the constant m will $u = e^{mx}$ be a solution to the differential equation

$$y'' - 3y' - 10y = 0$$

   [*10 points*]

5. Find the solution to the initial value problem

$$\frac{dz}{dt} = z + zt^2 \quad z = 5 \text{ when } t = 0$$

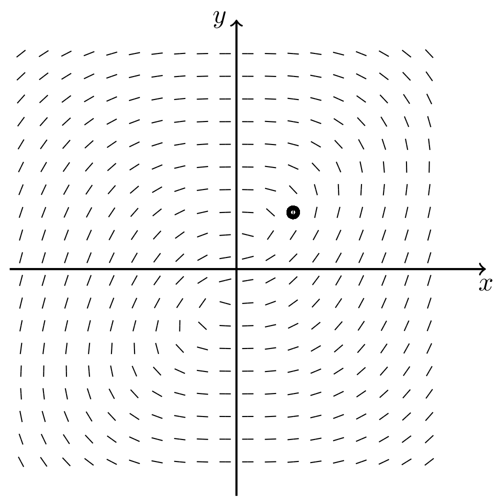
   [*15 points*]

6. The rate of growth of a tumor is proportional to the size of a tumor.
   (a) Write a differential equation satisfied by S, the size of the tumor, in mm, as a function of time t. Assume the proportionality constant, k, is positive.
   (b) Find the general solution to the differential equation.

7. The slope field for the differential equation

$$\frac{dy}{dx} = f(x, y)$$

is shown below. [*15 points*]



(a) Plot the solution that passes through the dot.

(b) Are there any equilibrium solutions? Yes or No.

(c) Match the related differential equation with its slope field below.

i. $\dfrac{dy}{dx} = -f(x, y)$

ii. $\dfrac{dy}{dx} = \dfrac{1}{f(x, y)}$