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Problem number:	1	2	3	4	5	6	7
Points available:	15	20	10	10	15	15	15

1. Find the third-order Taylor polynomial of  $(1+x)e^x$  about the point  $x=0$ . **Compute the Taylor polynomial directly from the definition - do not use the table of known Taylor series.**

Use the result above to calculate the exact value of the sum

$$\sum_{n=0}^{\infty} (-1)^n \frac{n+1}{n!}.$$

2. Padé approximants are rational functions used to approximate more complicated functions. In this problem, you will derive the Padé approximate to the exponential function.

(a) Let  $f(x) = (1+ax)/(1+bx)$ , where  $a$  and  $b$  are constants. Use any method you choose to write down the first four terms in the Taylor series for  $f(x)$  about  $x=0$ .

(b) What is the radius of convergence of the Taylor Series in (a)?

(c) Write down the first four terms in the Taylor series of  $g(x) = e^x$  about  $x=0$ .

(d) Use your results from (a) and (c) to explain why selecting  $a = 1/2$  and  $b = -1/2$  makes  $f(x)$  the best possible Padé approximate to  $g(x)$  near  $x=0$ .

(e) Write **one** sentence explaining how you could provide a reasonable estimate on the difference,  $f(x) - g(x)$ , for values of  $x$  that are very close to 0.

3. Identify the following series as the series expansion of a particular function and use this information to find the sum of the series.

$$(\sqrt{e}-1) - \frac{(\sqrt{e}-1)^2}{2} + \frac{(\sqrt{e}-1)^3}{3} + \dots$$

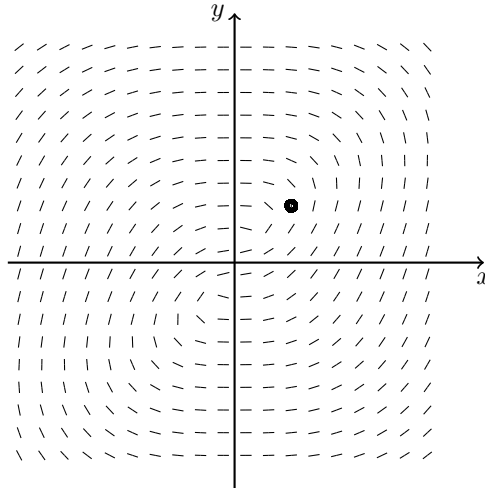
4. For what value(s) of the constant  $m$  will  $y = e^{mx}$  be a solution to the differential equation?

$$y'' - 3y' - 10y = 0$$

5. The slope field for the differential equation

$$\frac{dy}{dx} = f(x, y)$$

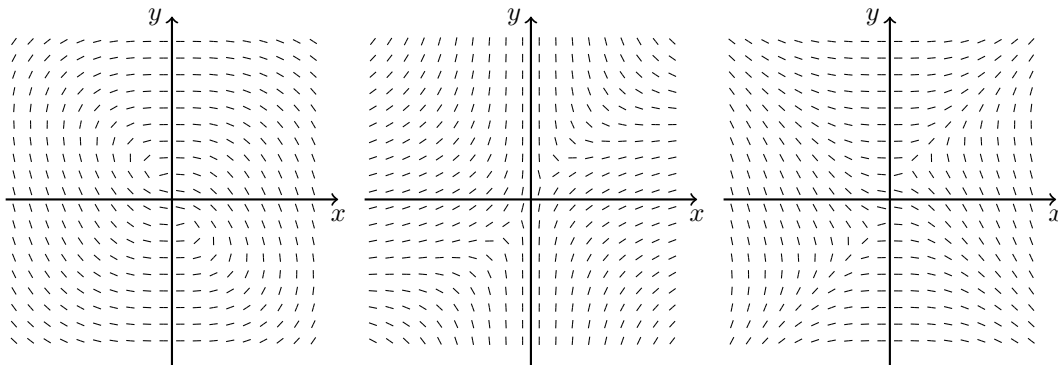
is shown below.



- (a) Plot the solution that passes through the dot.
- (b) Are there any equilibrium solutions? Yes or No.
- (c) Match the related differential equation with its slope field below.

(i)  $\frac{dy}{dx} = -f(x, y)$

(ii)  $\frac{dy}{dx} = \frac{1}{f(x, y)}$



6. Find the solution to the initial value problem

$$\frac{dz}{dt} = z + zt^2 \quad z = 5 \text{ when } t = 0.$$

7. The rate of growth of a tumor is proportional to the size of a tumor.

- (a) Write a differential equation satisfied by  $S$ , the size of the tumor, in mm, as a function of time  $t$ . Assume the proportionality constant,  $k$ , is positive.
- (b) Find the general solution to the differential equation.