

1. [5 points] Circle all solutions to the differential equation

$$y'' - 5y' + 6y = 0$$

- a) $y = e^{2x}$
- b) $y = x^2$
- c) $y = e^{3x}$

2. [5 points] Suppose the Taylor series about $x = 1$ for a function f is

$$f(x) = \sum_{n=0}^{\infty} \frac{2^n}{(2n)!} (x-1)^{2n}$$

What is $f^{(4)}(1)$?

- a) 1
 - b) 2
 - c) 4
 - d) 8
 - e) None of these
3. [10 points] Use Taylor series to find the exact sum of the series

$$1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \dots$$

4. [15 points] Write out the first four nonzero terms for the Taylor series of the function for x near 0. You may use the table provided.

$$g(x) = \frac{\sin(x)}{1+x}$$

5. [15 points] Write out the first five terms of the Taylor series for \sqrt{x} centered at $x = 1$. **Compute the terms directly from the definition - do not use the table of known Taylor series.**

6. [15 points] Solve the initial value problem:

$$\frac{dw}{dz} = -w^2 \tan(z), \quad w(0) = 2.$$

7. [15 points] A thermometer is taken from a room that is 20°C to the outdoors where the temperature is 40°C . After one minute, the thermometer reads 23°C . Such a situation can be described by the differential equation

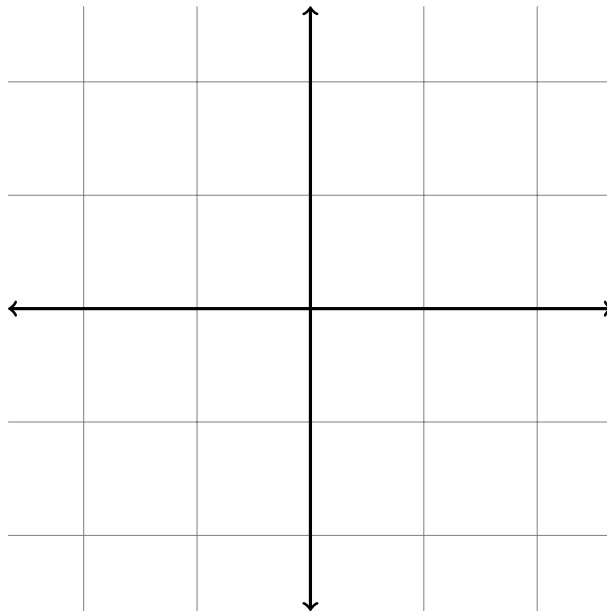
$$\frac{dT}{dt} = k(40 - T), \quad T(0) = 20, \quad T(1) = 23.$$

- (a) What will the reading on the thermometer be after one more minute?
 (b) When will the thermometer read 35°C ?

8. [20 points] Consider the differential equation

$$\frac{dy}{dx} = -\left(y - \frac{1}{2}\right)\left(y - \frac{3}{2}\right)(y + 1)$$

- a) Draw the slope field for this equation. (It does not need to be meticulously detailed, but it should include the important features!)



- b) Using the slope field, find and classify (as stable or unstable) all equilibrium solutions to the differential equation.