Math 129 Calculus II   Exam # 3, Sections 8.5 & 9.1 - 9.5   Time allowed: 60 minutes

Problem number:	1	2	3	4	5	6	7
Points available:	20	20	15	15	15	10	5

1. A cylindrical barrel, standing upright on its circular end, contains muddy water. The top of the barrel, which has a diameter of 1 meter, is open. The height of the barrel is 1.8 meters and it is filled to a depth of 1.5 meters. The density of the muddy water at a depth h meters below the surface is given by $\delta(h) = 1 + kh$ (units are $\frac{kg}{m^3}$) where k is some positive constant. Set up the integral to find the total work done to pump the muddy water to the top of the rim of the barrel. **You do not need to evaluate the integral.** Leave your answer in terms of π, g (the gravitational constant, units are $\frac{m}{s^2}$), and k.

2. Determine if each of the following converges or diverges. You must justify your answer. For the series, do this by clearly stating which test you use and show that all requirements of that test are met.

   (a) The **sequence** $a_n = \dfrac{(-1)^n n^3}{n^3 + 2n^2 + 1}$

   (b) The **series** $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7 + n^2}}$

   (c) The **series** $1 - 10 + \dfrac{100}{2} - \dfrac{1000}{3!} + \dfrac{10000}{4!} - \dots$

3. Recall that we proved in class that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = 1$. You may use this fact to solve the following problem.

   Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \left(\frac{3}{n(n+1)} + \frac{1}{2^n} \right)$.

4. Prove that the series $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n^2}$ converges. Justify your work by clearly stating which test you use and show that all requirements of that test are met.

5. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{n}$.

   (a) Find the radius of convergence of the series.

   (b) Give the interval of convergence (do not worry about checking the endpoints).

6. Mark each as true or false:

   (a) ______ The convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$ can be investigated using the Alternating Series Test.

   (b) ______ If $\sum c_n x^n$ diverges when $x = 6$, then it diverges when $x = 10$.

   (c) ______ If $0 \le a_n \le b_n$ and $\sum b_n$ diverges, then $\sum a_n$ diverges.

   (d) ______ If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} < \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges.

   (e) ______ If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

7. Suppose the partial sums of a series are given by the equation

$$S_n = \frac{13n+1}{n}.$$

   Choose one of the following options:

   (a) The series converges.

   (b) The series diverges by the Integral Test.

   (c) The series diverges by the Limit Comparison Test with $\sum_{n=1}^{\infty} \frac{1}{n}$.

   (d) The series diverges by the Ratio Test.