Fall 2016

Math 129 - 01 Calculus II

Exam # 3, Sections 8.5 - 9.5

Time allocated: 60 minutes

Calculators: Permitted

Instructor's Name: Colin Clark

Student's Name (please print): ____________________

By signing my name below, I agree that I am following all rules and regulations set forth by the Code of Academic Integrity. Furthermore, I agree that I am following all rules set by my instructor and by the course policy for this exam. This includes ensuring that all calculator programs have been deleted.

Signature:____________________Date:____________________

1. [**15 points**] Determine whether the following are true or false.

_____ The convergence or divergence of the series below can be investigated using the Alternating series test.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3n}{4n-1}$$

_____ If $p_n \leq q_n \leq 0$ for each n and $\sum_{n=1}^{\infty} p_n$ converges, then $\sum_{n=1}^{\infty} q_n$ converges.

(Hint: Draw a picture!)

_____ If $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} < \infty$ then the series $\sum_{n=1}^{\infty} a_n$ converges.

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2. [**5 points**] Consider the series below.

$$5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$$

To what value does the series converge?

(a) $-\frac{2}{3}$

(b) 0

(c) $\frac{2}{3}$

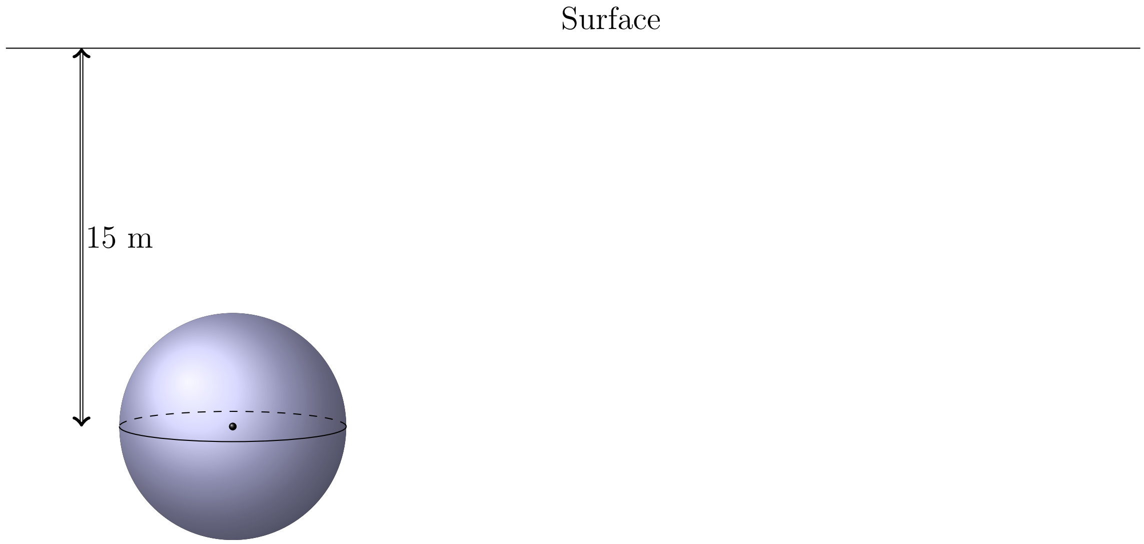
(d) 3

(e) 5

(f) The series diverges

3. [**20 points**] A spherical tank of radius 5 meters is buried in the ground. Suppose the center of the tank is 15 meters below the surface and the tank is completely full of water which has density 1000 kg/m^3.

a) Draw a slice (taken parallel to the ground) of water h meters from the center of the tank with a thickness Δh. Give an expression for the work required to pump the slice to the surface in terms of h and Δh. Use $g = 9.8$ m/s^2.



b) Use your result from part a to write an integral which gives the total work required to pump water from the tank to the surface. You do not need to solve this integral.

4. [**20 points**] Use the integral test to determine whether the series below converges. Justify your work by showing that all requirements of the test are met.

$$\sum_{j=3}^{\infty} \frac{j^2}{e^j}$$

5. [**20 points**] Determine whether the series is convergent or divergent. Justify your answer by clearly stating which test you use and by showing that all requirements of the test are met.

$$\sum_{n=1}^{\infty} \frac{2\sqrt{n}}{n^2 - 4n + 5}$$

6. [**20 points**] Determine whether the series is convergent or divergent. Justify your answer by clearly stating which test you use and by showing that all requirements of the test are met.

$$\sum_{n=2}^{\infty} \frac{3^n}{(n-1)!}$$

What can you say about the series

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{(n-1)!}$$

7. [**20 points**] Determine whether the series below converges when $x = 4$. Justify your answer by clearly stating which test you use and by showing that all requirements of the test are met.

$$\sum_{n=0}^{\infty} \frac{2n(x+2)^n}{5^n}.$$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int a^x\,dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\,dx = x\ln x - x + C, \quad x > 0$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \tan x\,dx = -\ln|\cos x| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\int x^n \ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, \quad x > 0$
14. $\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}\,dx$
    $= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots$
    $(+-+-\ldots)$ (signs alternate)
15. $\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\,dx$
    $= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$
    $(-++--++\ldots)$ (signs alternate in pairs after first term)
16. $\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$
    $= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$
    $(++--++--\ldots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx, \quad n$ positive
18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx, \quad n$ positive
19. $\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx, \quad m \neq 1, m$ positive
20. $\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + C$
21. $\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx, \quad m \neq 1, m$ positive
22. $\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + C$
23. $\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
25. $\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$
27. $\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C, \quad a \neq b$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$
29. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$
31. $\int \sqrt{x^2 - a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2 - a^2} - a^2\int \frac{1}{\sqrt{x^2 - a^2}}\,dx\right) + C$