
Math 129 - Practice Test 2

1) Evaluate the following integral or show that it is divergent.

$$\int_1^{\infty} \frac{1}{\sqrt{2x+1}} dx$$

2) Evaluate the following integral or show that it is divergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}} dx$$

3) Determine if the improper integral converges or diverges. Justify your answer.

$$\int_0^{\infty} \frac{\arctan x}{2 + e^x} dx$$

4) Find the arc length of the curve $y = \ln(\cos x)$ for $0 \leq x \leq \frac{\pi}{3}$.

5) Set up the integral to find the volume of the solid obtained by rotating the region bounded by the curves $x = 0$ and $x = 9 - y^2$ about $x = -1$. You do not have to solve the integral.

6) Find the volume of the solid S if the base of S is the triangular region with vertices at $(0, 0)$, $(3, 0)$, and $(0, 3)$ and cross sections perpendicular to the x -axis are semicircles.

7) A circular disk of radius 6 centimeters is constructed so that the density of the metal that makes up the disk is given by the function $\delta(x) = \sqrt{100 - x^2}$ g/cm², where x is the distance from the center of the disk. Find the total mass of the disk. Leave your answer in exact form.

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n dx = \frac{1}{n+1}x^{n+1} + C, n \neq -1$
2. $\int \frac{1}{x} dx = \ln|x| + C$
3. $\int a^x dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x dx = x \ln x - x + C, x > 0$
5. $\int \sin x dx = -\cos x + C$
6. $\int \cos x dx = \sin x + C$
7. $\int \tan x dx = -\ln|\cos x| + C$

II. Products of e^x , $\cos x$, and $\sin x$

8. $\int e^{ax} \sin(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \sin(bx) - b \cos(bx)] + C$
9. $\int e^{ax} \cos(bx) dx = \frac{1}{a^2 + b^2} e^{ax} [a \cos(bx) + b \sin(bx)] + C$
10. $\int \sin(ax) \sin(bx) dx = \frac{1}{b^2 - a^2} [a \cos(ax) \sin(bx) - b \sin(ax) \cos(bx)] + C, a \neq b$
11. $\int \cos(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \cos(ax) \sin(bx) - a \sin(ax) \cos(bx)] + C, a \neq b$
12. $\int \sin(ax) \cos(bx) dx = \frac{1}{b^2 - a^2} [b \sin(ax) \sin(bx) + a \cos(ax) \cos(bx)] + C, a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x , $\cos x$, $\sin x$

13. $\int x^n \ln x dx = \frac{1}{n+1} x^{n+1} \ln x - \frac{1}{(n+1)^2} x^{n+1} + C, n \neq -1, x > 0$
14. $\int p(x) e^{ax} dx = \frac{1}{a} p(x) e^{ax} - \frac{1}{a} \int p'(x) e^{ax} dx$
 $= \frac{1}{a} p(x) e^{ax} - \frac{1}{a^2} p'(x) e^{ax} + \frac{1}{a^3} p''(x) e^{ax} - \dots$
 (+ - + - ...) (signs alternate)
15. $\int p(x) \sin ax dx = -\frac{1}{a} p(x) \cos ax + \frac{1}{a} \int p'(x) \cos ax dx$
 $= -\frac{1}{a} p(x) \cos ax + \frac{1}{a^2} p'(x) \sin ax + \frac{1}{a^3} p''(x) \cos ax - \dots$
 (- + - + ...) (signs alternate in pairs after first term)
16. $\int p(x) \cos ax dx = \frac{1}{a} p(x) \sin ax - \frac{1}{a} \int p'(x) \sin ax dx$
 $= \frac{1}{a} p(x) \sin ax + \frac{1}{a^2} p'(x) \cos ax - \frac{1}{a^3} p''(x) \sin ax - \dots$
 (+ - + - ...) (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx, n$ positive
18. $\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx, n$ positive
19. $\int \frac{1}{\sin^m x} dx = \frac{-1}{m-1} \frac{\cos x}{\sin^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\sin^{m-2} x} dx, m \neq 1, m$ positive
20. $\int \frac{1}{\cos x} dx = \frac{1}{2} \ln \left| \frac{\cos x - 1}{\cos x + 1} \right| + C$
21. $\int \frac{1}{\cos^m x} dx = \frac{1}{m-1} \frac{\sin x}{\cos^{m-1} x} + \frac{m-2}{m-1} \int \frac{1}{\cos^{m-2} x} dx, m \neq 1, m$ positive
22. $\int \frac{1}{\cos x} dx = \frac{1}{2} \ln \left| \frac{\sin x + 1}{\sin x - 1} \right| + C$
23. $\int \sin^m x \cos^n x dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is omitted.

V. Quadratic In the Denominator

24. $\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan \frac{x}{a} + C, a \neq 0$
25. $\int \frac{bx + c}{x^2 + a^2} dx = \frac{b}{2} \ln|x^2 + a^2| + \frac{c}{a} \arctan \frac{x}{a} + C, a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)} dx = \frac{1}{a-b} (\ln|x-a| - \ln|x-b|) + C, a \neq b$
27. $\int \frac{1}{(x-a)(x-b)} dx = \frac{1}{a-b} [(ac+d) \ln|x-a| - (bc+d) \ln|x-b|] + C, a \neq b$

VI. Integrands Involving $\sqrt{a^2 + x^2}$, $\sqrt{a^2 - x^2}$, $\sqrt{x^2 - a^2}$, $a > 0$

28. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C$
29. $\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$
30. $\int \sqrt{a^2 \pm x^2} dx = \frac{1}{2} (x\sqrt{a^2 \pm x^2} + a^2 \int \frac{1}{\sqrt{a^2 \pm x^2}} dx) + C$
31. $\int \sqrt{x^2 - a^2} dx = \frac{1}{2} (x\sqrt{x^2 - a^2} - a^2 \int \frac{1}{\sqrt{x^2 - a^2}} dx) + C$