Math 129 - Calculus II Exam # 2, Sections 7.6 - 7.7 & 8.1 - 8.4 Time allowed: 60 minutes

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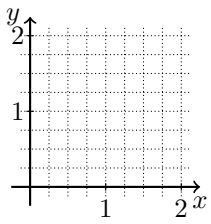
1. Evalulate the following integral exactly or show that it diverges. You must show all your work and use the correct notation. [*15 points*]

$$\int_1^\infty \frac{1}{4s^2+1}\,ds$$

2. Use the comparison test to determine whether the following improper integral converges or diverges. If the integral converges, give a bound on it's value. If it diverges, show that it diverges. [*15 points*]

$$\int_2^\infty \frac{1}{t^{1/2}-1}\,dt$$

3. (a) Use the fact that $e^t > 1+t$ to sketch $y = \dfrac{1}{e^x-1}$ and $y = \dfrac{1}{x}$ on the same axes.



   (b) Use your sketch to select which of the following statements is correct. There is only one correct answer.

   A. The comparison test shows that $\displaystyle\int_0^1 \frac{1}{e^x-1}\,dx$ converges.

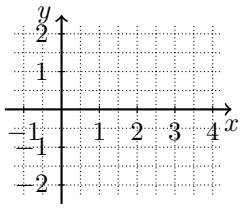
   B. The comparison test shows that $\displaystyle\int_0^1 \frac{1}{e^x-1}\,dx$ diverges.

   C. The comparison test is inconclusive.

   (c) Write two sentences (referring to your diagram if necessary) to defend your answer in (b).

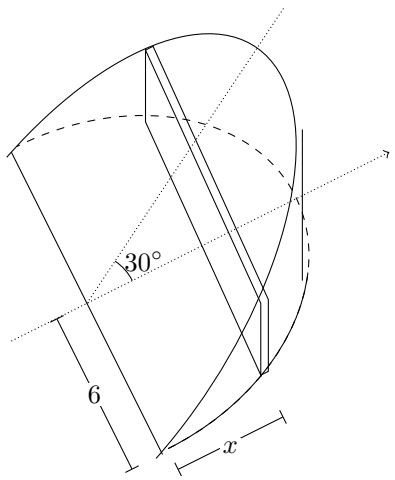
4. In this problem, we will investigate the arc length of the curve $y = 2\sin(x^2+1)$ between $x = 1$ and $x = 4$. Make sure your calculator is in radians. [*15 points*]

   (a) Draw a sketch of the curve $y = 2\sin(x^2+1)$ to determine whether the arc length of the curve will be greater than or less than 16.



   (b) Write down an integral that expresses the arc length of the curve but do not evaluate the integral.

5. The solid in the figure below is created by cutting a wedge from a half-cylinder of radius 6cm. The solid has a slanted roof with angle of inclination of 30°.



Write down an integral that expresses the volume of the solid. Show all your work, clearly indicating the shape and dimensions of your slices. Use diagrams to support your calculations. Do not evaluate your integral. [*15 points*]

6. Let R be the region bounded by the curves $y = \dfrac{1}{x}$ and $y = 3 - x$. [*15 points*]

   (a) Make an accurate sketch of the region R.

   (b) Set up, but do not evaluate, the integral that gives the area of this region.

   (c) Set up, but do not evaluate, the integral that gives the volume of the solid obtained when R is rotated about the line $y = -1$.

7. The density of air h meters above the earth's surface is approximately $\delta(h) = 1.2e^{-h/5000}$, where δ has units kg/m^3. Find the mass of a cylindrical column of air 10 meters in diameter and 6000 meters high. The base of the cylindrical column is on the surface of the earth at sea level. [*15 points*]