

Problem number:	1	2	3	4	5	6	7	8
Points available:	10	15	5	10	15	15	15	15

1. Determine whether the following integrals are proper or improper. Mark your answer with an X.

$$\int_0^3 \frac{1}{x-1} dx \quad \text{_____ proper} \quad \text{_____ improper}$$

$$\int_0^1 \frac{1}{x^2+1} dx \quad \text{_____ proper} \quad \text{_____ improper}$$

2. Suppose  $f$  and  $g$  are continuous functions. If  $0 < f(x) < \frac{1}{x^4}$  and  $\int_0^\infty 3g(x) dx$  converges, determine if the following improper integrals converge or diverge. Mark your answer with an X.

$$\int_0^1 f(x) dx \quad \text{_____ converges} \quad \text{_____ diverges} \quad \text{_____ impossible to tell}$$

$$\int_7^\infty g(x) dx \quad \text{_____ converges} \quad \text{_____ diverges} \quad \text{_____ impossible to tell}$$

$$\int_0^\infty g(x-3) dx \quad \text{_____ converges} \quad \text{_____ diverges} \quad \text{_____ impossible to tell}$$

3. Which of the following gives the arc length of the curve  $f(x) = \sqrt{x}$  from  $x = 1$  to  $x = 7$ ? Mark your answer with an X.

$$\text{_____ } \int_1^7 \frac{\sqrt{4x+1}}{2\sqrt{x}} dx$$

$$\text{_____ } \int_1^7 \sqrt{\frac{1}{2\sqrt{x}}} dx$$

$$\text{_____ } \int_1^7 \sqrt{1 + \frac{1}{2x}} dx$$

$$\text{_____ } \int_1^7 \sqrt{1 + \frac{1}{2\sqrt{x}}} dx$$

4. Write down definite integrals which give the area underneath the curve  $y = e^{2x}$ , above the  $x$ -axis, and between  $x = -1$  and  $x = 5$  by slicing as instructed. **Do not evaluate the integrals.**

(a) Vertical slices

(b) Horizontal slices

5. Write down a definite integral (include all your work to get there) which gives the volume of the solid constructed in the following way: the base is the region between the curves  $y = 5 - x$ ,  $y = 0$ , and  $x = 0$ , and the cross-sections perpendicular to the  $y$ -axis are squares. **Do not evaluate the integral.**

6. Evaluate the integral below or prove that it diverges. **The point of this problem is to demonstrate your understanding of improper integrals by treating every step with care, using correct notation, and fully justifying your work.**

$$\int_{-\infty}^0 x e^x dx$$

7. Consider the region bounded by the  $x$ -axis, the line  $x = 1$ , and the curve  $y = \frac{1}{x}$ .

(a) Sketch this region accurately.

(b) Show that the region has infinite area.

(c) Show that the solid of revolution formed by rotating the region about the  $x$ -axis is finite - you can show this by calculating the volume and seeing that it is a finite number.

8. Find the total mass of the triangular region shown, which has density  $\delta(x) = 1 + 2x \text{ g/cm}^2$ .

