Math 129 - Practice Test 1

1) Decide which of the following techiniques would be used to evaluate the integral, but do not evaluate the integral.

$$\int \sqrt{3-2x-x^2}dx$$

Select all that apply.

A) Substitution

B) Integration by parts

C) Partial Fractions

D) Table of integrals

E) Trigonometric substitution

2) Decide which of the following techiniques would be used to evaluate the integral, but do not evaluate the integral.

$$\int \frac{4t}{t^2-9}dt$$

Select all that apply.

A) Substitution

B) Integration by parts

C) Partial Fractions

D) Table of integrals

E) Trigonometric substitution

3) Evaluate the following integral.

$$\int_{\pi/6}^{\pi/2} \cos x \ln(\sin x) dx$$

4) Evaluate the following integral.

$$\int \frac{\sqrt{x^2 - 9}}{x^3} dx$$

5) Evaluate the following integral.

$$\int \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2} dx$$

6) Suppose that $\int_0^9 f(u)du = 4$ and $f(9) = 2$, use integration by parts to evaluate the following definite integral.

$$\int_0^3 uf'(3u)du$$

7) i) Use the trapezoid rule with $n = 3$ to approximate the following integral. (Round your answer to 5 decimal places)

$$\int_0^{0.3} \sin(x^2)dx$$

ii) Is your estimate in i) an overestimate or an underestimate? Explain in a sentence.

iii) If you were to instead use the midpoint method, would you answer be larger or smaller than your answer in i)? Explain in a sentence.

8) Evaluate the following integral.

$$\int (2y^2 - 3)\cos(3y)dy$$

A Short Table of Indefinite Integrals

I. Basic Functions

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1$
2. $\int \frac{1}{x}\,dx = \ln|x| + C$
3. $\int a^x\,dx = \frac{1}{\ln a}a^x + C$
4. $\int \ln x\,dx = x\ln x - x + C, \quad x > 0$
5. $\int \sin x\,dx = -\cos x + C$
6. $\int \cos x\,dx = \sin x + C$
7. $\int \tan x\,dx = -\ln|\cos x| + C$

II. Products of e^x, $\cos x$, and $\sin x$

8. $\int e^{ax}\sin(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\sin(bx) - b\cos(bx)] + C$
9. $\int e^{ax}\cos(bx)\,dx = \frac{1}{a^2+b^2}e^{ax}[a\cos(bx) + b\sin(bx)] + C$
10. $\int \sin(ax)\sin(bx)\,dx = \frac{1}{b^2-a^2}[a\cos(ax)\sin(bx) - b\sin(ax)\cos(bx)] + C, \quad a \neq b$
11. $\int \cos(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\cos(ax)\sin(bx) - a\sin(ax)\cos(bx)] + C, \quad a \neq b$
12. $\int \sin(ax)\cos(bx)\,dx = \frac{1}{b^2-a^2}[b\sin(ax)\sin(bx) + a\cos(ax)\cos(bx)] + C, \quad a \neq b$

III. Product of Polynomial $p(x)$ with $\ln x$, e^x, $\cos x$, $\sin x$

13. $\int x^n \ln x\,dx = \frac{1}{n+1}x^{n+1}\ln x - \frac{1}{(n+1)^2}x^{n+1} + C, \quad n \neq -1, \quad x > 0$
14. $\int p(x)e^{ax}\,dx = \frac{1}{a}p(x)e^{ax} - \frac{1}{a}\int p'(x)e^{ax}\,dx$
$= \frac{1}{a}p(x)e^{ax} - \frac{1}{a^2}p'(x)e^{ax} + \frac{1}{a^3}p''(x)e^{ax} - \cdots$
$(+-+-\ldots)$ (signs alternate)
15. $\int p(x)\sin ax\,dx = -\frac{1}{a}p(x)\cos ax + \frac{1}{a}\int p'(x)\cos ax\,dx$
$= -\frac{1}{a}p(x)\cos ax + \frac{1}{a^2}p'(x)\sin ax + \frac{1}{a^3}p''(x)\cos ax - \cdots$
$(-++--++\ldots)$ (signs alternate in pairs after first term)
16. $\int p(x)\cos ax\,dx = \frac{1}{a}p(x)\sin ax - \frac{1}{a}\int p'(x)\sin ax\,dx$
$= \frac{1}{a}p(x)\sin ax + \frac{1}{a^2}p'(x)\cos ax - \frac{1}{a^3}p''(x)\sin ax - \cdots$
$(++--++--\ldots)$ (signs alternate in pairs)

IV. Integer Powers of $\sin x$ and $\cos x$

17. $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2}x\,dx, \quad n$ positive
18. $\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx, \quad n$ positive
19. $\int \frac{1}{\sin^m x}\,dx = \frac{-1}{m-1}\frac{\cos x}{\sin^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\sin^{m-2}x}\,dx, \quad m \neq 1, m$ positive
20. $\int \frac{1}{\sin x}\,dx = \frac{1}{2}\ln\left|\frac{(\cos x)-1}{(\cos x)+1}\right| + C$
21. $\int \frac{1}{\cos^m x}\,dx = \frac{1}{m-1}\frac{\sin x}{\cos^{m-1}x} + \frac{m-2}{m-1}\int \frac{1}{\cos^{m-2}x}\,dx, \quad m \neq 1, m$ positive
22. $\int \frac{1}{\cos x}\,dx = \frac{1}{2}\ln\left|\frac{(\sin x)+1}{(\sin x)-1}\right| + C$
23. $\int \sin^m x\cos^n x\,dx$: If m is odd, let $w = \cos x$. If n is odd, let $w = \sin x$. If both m and n are even and non-negative, convert all to $\sin x$ or all to $\cos x$ (using $\sin^2 x + \cos^2 x = 1$), and use IV-17 or IV-18. If m and n are even and one of them is negative, convert to whichever function is in the denominator and use IV-19 or IV-21. The case in which both m and n are even and negative is omitted.

V. Quadratic in the Denominator

24. $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
25. $\int \frac{bx+c}{x^2+a^2}\,dx = \frac{b}{2}\ln|x^2+a^2| + \frac{c}{a}\arctan\frac{x}{a} + C, \quad a \neq 0$
26. $\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}(\ln|x-a| - \ln|x-b|) + C, \quad a \neq b$
27. $\int \frac{cx+d}{(x-a)(x-b)}\,dx = \frac{1}{a-b}[(ac+d)\ln|x-a| - (bc+d)\ln|x-b|] + C, \quad a \neq b$

VI. Integrands Involving $\sqrt{a^2+x^2}$, $\sqrt{a^2-x^2}$, $\sqrt{x^2-a^2}$, $a > 0$

28. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \arcsin\frac{x}{a} + C$
29. $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C$
30. $\int \sqrt{a^2 \pm x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 \pm x^2} + a^2\int \frac{1}{\sqrt{a^2 \pm x^2}}\,dx\right) + C$
31. $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}\left(x\sqrt{x^2-a^2} - a^2\int \frac{1}{\sqrt{x^2-a^2}}\,dx\right) + C$