

1. Which of the following techniques could be used to evaluate the following integral

$$\int \frac{x}{\sqrt{25-x^2}} dx.$$

Circle all that apply. [12 points]

- (a) Substitution: Let  $w = 25 - x^2$ .
- (b) Integration by parts: Let  $u = \frac{1}{\sqrt{25-x^2}}$ ,  $v' = x$
- (c) Integration by parts: Let  $u = x$ ,  $v' = \frac{1}{\sqrt{25-x^2}}$
- (d) Partial Fractions: Decomposition structure given by  $\frac{A}{5-x} + \frac{B}{5+x}$
- (e) Trigonometric Substitution: Let  $x = 5 \sin(\theta)$
- (f) Trigonometric Substitution: Let  $x = 5 \tan(\theta)$

2. Circle the answer corresponding to the partial fraction expansion of

$$\frac{x^4 - 1}{x(x^2 + 4)}$$

Note: There is only one correct answer. [3 points]

- (a)  $\frac{A}{x} + \frac{Bx + C}{x^2 + 4}$
- (b)  $\frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+4}$
- (c)  $x + \frac{A}{x} + \frac{Bx + C}{x^2 + 4}$
- (d)  $x + \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+4}$

3. Evaluate the definite integral

$$\int_0^{\pi^2} \frac{\sin\left(\frac{1}{4}\sqrt{x}\right)}{3\sqrt{x}} dx$$

[15 points]

4. If  $f$  is a twice differentiable function, find an expression for

$$\int f''(x) \ln(x) dx + \int \frac{f(x)}{x^2} dx.$$

(Your answer should contain  $f$  and  $f'$  but no integrals.) [15 points]

5. (a) Peyton Manning wants to calculate the integral below without using the table.

$$\int (5x^2 - 8) e^{x/2} dx$$

Do **not** evaluate the integral, but instead, recommend a strategy for Mr. Manning and write **one** sentence explaining what he should expect to see. [3 points]

(b) Use the table to evaluate the integral from (a). [12 points]

6. Calculate the following definite integral without using the tables.

$$\int_1^2 \frac{x^3 + 25}{x^2(x - 5)} dx.$$

[15 points]

7. Use an appropriate trigonometric substitution to calculate the integral

$$\int x^4 \sqrt{1 - 4x^2} dx.$$

[15 points]

8. (a) Use the trapezoid rule with three subdivisions to estimate the value of the integral

$$\int_0^1 \sin(e^x) dx.$$

[5 points]

(b) Is your answer an overestimate or an underestimate? Explain. [2 points]

(c) Cam Newton sees your work in part (a) and suggests using three subdivisions to evaluate the integral

$$\int_2^3 \sin(e^x) dx.$$

Write **one** sentence to explain to Mr. Newton why this is not a good idea. (Hint: Draw a sketch.) [3 points]