Math 116: Business Calculus
Chapter 4 - Calculating Derivatives

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Exam 2 - Thursday April 27.

6.1 Absolute Extrema.

6.2 Applications of Extrema.

6.3 Further Applications.

7.1 Anti-derivatives.

7.2 Substitution.

7.3 Area and Definite Integral.

7.4 Three Fundamental Theorems.

7.5 Area between Two Curves.

8.3 Continuous Mondey Flow.

Example 1. Absolute Extrema

Terminology

1. Absolute / global extrema vs. Relative exrema
2. Absolute / global minima vs. Relative minima
3. Absolute / global maxima vs. Relative maxima

Terminology

Let f be a function defined on some interval. Let c be a number in the interval. Then $f(c)$ is the **absolute maximum** of f on the interval if

$$f(x) \leq f(c)$$

for every x in the interval. and $f(c)$ is the absolute minumum of f on the interval if

$$f(x) \geq f(c)$$

for every x in the interval.

Extreme Value Theorem

Every function$f$ that is continuouis on a closed interval $[a, b]$ will have both an absolute maximum and an absolute minumum on the interval.

Finding Absolute Extrema

To find absolute extrema for a functio f, continuous on a closed interval $[a, b]$

1. Find all critical numbers for f in (a, b).
2. Evaluate f at all critical numbers in (a, b).
3. Evaluate f for the endpoints a and b of the interval $[a, b]$.
4. The largest value found in step 2 or 3 is the absolute maximum for f on $[a, b]$ and the smallest value found is the absolute minimum for f on $[a, b]$.

Example 1. Absolute Extrema

Find the absolute extrema of the function

$$f(x) = x^{8/3} - 16x^{2/3}$$

on the interval $[-1,8]$.

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$$f(x) = x^{8/3} - 16x^{2/3}$$

on the interval $[-1, 8]$. Absolute minimum is -19.05 at $x = 2$. Absolute maximum is 192 at x = 8.

Example 2. Absolute Extrema

Find the locations and values of the absolute extrema, if they exist, for the function

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 2.$$

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Absolute mininum is -30 at $x = 2$. Absolute maximum is 2 at x = 0.

Example 3. Dollar Exchange

The U.S. and Canadian exchange rate changes daily. The value of the U.S. dollar (in Canadian dolalrs) between 2010 and 2014 can be approximated by the function

$$f(t) = -0.00269t^3 + 0.03610t^2 - 0.1003t + 1.0645$$

where t is the number of years since 2010. Based on this approximation, in what year during this period did the value of the U.S. dollar reach its absolute minimum? What was the minimum?

Problem 8. Maximization

The sale of CDs of ‘lesser’ performers is very sensitive to price. If a CD manufacturer charges $p(x)$ dollars per CD, where if

$$p(x) = 12 - \frac{x}{8}$$

then x thousand CDs will be sold.

1. Find an expression for the total revenue from the sale of x thousand CDs.
2. Find the value of x that leads to maximum revenue.
3. Find the maximum revenue.

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1. Find an expression for the total revenue from the sale of x thousand CDs. $R(x) = 12{,}000x - 125x^2$
2. Find the value of x that leads to maximum revenue.48,000 CDs
3. Find the maximum revenue. \$288,000

Problem 5. Minimizing Time

Determine the average cost function, and find the production level that gives the minimum average cost.

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$\overline{C}(x) = x^2/2 + 2x - 3 + 35/x$
$x = 2.722$

Problem 15. Maximizing Volume

A local club is arranging a charter flight to Hawaii. The cost of the trip is \$1,600 each for 90 passengers, with a refund of \$10 per passenger for each passenger in excess of 90.

1. Find the nubmer of passengers that will maximize the revenue received from the flight.
2. Find the maximum revenue.

Problem 15. Maximizing Volume

A local club is arranging a charter flight to Hawaii. The cost of the trip is $1,600 each for 90 passengers, with a refund of $10 per passenger for each passenger in excess of 90.

1. Find the nubmer of passengers that will maximize the revenue received from the flight. 125 passengers.
2. Find the maximum revenue.$156,250

Problem 16. Maximizing Volume

In planning a restaurant, it is estimated that a profit of $8 per seat will be made if the number of seats is no more than 50, inclusive. On the other hand, the profit on each seat will decrease by 10c for each seat above 50.

1. Find the number of seats that will produce the maximum profit.
2. What is the maximum profit.

Problem 16. Maximizing Volume

In planning a restaurant, it is estimated that a profit of $8 per seat will be made if the number of seats is no more than 50, inclusive. On the other hand, the profit on each seat will decrease by 10c for each seat above 50.

1. Find the number of seats that will produce the maximum profit. 65 seats.
2. What is the maximum profit. $422.50

Elasiticity of Demand

Let $q = f(p)$, where q is demand at a price p. The elasticity of demand is

$$E = -\frac{p}{q} \cdot \frac{dq}{dp}$$

1. Demand is inelastic if $E < 1$
2. Demand has unit elasticity if $E = 1$
3. Demand is elastic if $E > 1$

Example 3.

Terrence Wales described the demand for distilled psirits as

$$q = f(p) = -0.00375p + 7.87$$

where p represents the retail price of a case of liquor in dollars per case. Here q represents the average number of cases purchased per year by ta consumer. Calculate and interpret the elasticity of demand when $p = \$118.30$ per case.

Example 4.

The demand for beer was modeled by Hagarty and Elzinga with the function give by

$$q = f(p) = 1/p.$$

The price was expressed in dollars per can of beer, and the quantity sold in cans per day per adult. Calculate and interpret the elasticity of demand.

Revenue and Elasticity

1. If the demand is inelastic, total revenue increases as price increases.
2. Total revenue is maximized at the price where demand has unit elasticity.
3. If the demand is elastic, total revenue decreases as the price increases.

Example 5.

Assume that the demand for a pricut is $q = 216 - 2p^2$, where p is the price in dollars.

1. Find the price intervals where the demand is elastic and where demand is inelastic
2. What price results in the maximum revenue? What is the maximum revenue?