Math 116: Business Calculus
Chapter 4 - Calculating Derivatives

Instructor: Colin Clark

Spring 2017

Exam 2 - Thursday March 9.

4.1 Techniques for Finding Derivatives.

4.2 Derivatives of Products and Quotients.

4.3 The Chain Rule.

4.4 Derivatives of Exponential Functions.

4.5 Derivatives of Logarithmic Functions.

5.1 Increasing and Decreasing Functions.

5.2 Relative Extrema.

Example 1. Increasing and Decreasing

Terminology

Let f be a function defined on some interval. Then for any two numbers x_1 and x_2 in the interval, f is increasing on the interval if

$$f(x_1) < f(x_2) \quad \text{if } x_1 < x_2$$

and f is decreasing on the interval if

$$f(x_1) > f(x_2) \quad \text{if } x_1 < x_2$$

Terminology

Suppose a function f has a derivative at each point in an open interval; then

- if $f'(x) > 0$ for each x in the interval, f is increasing on the interval;
- if $f'(x) < 0$ for each x in the interval, f is decreasing on the interval;
- if $f'(x) = 0$ for each x in the interval, f is constant on the interval.

Critical Numbers

The critical numbers for a function f are those numbers c in the domain of f for which $f'(c) = 0$ or $f'(c)$ does not exist. A critical point is a point whose x-coordinate is the critical number c and whose y- coordinate $f(c)$.

Example 2. Increasing and Decreasing

Find the intervals in which the following function is increasing or decreasing. Locate all points where the tangent line is horizontal. Graph the function.

$$f(x) = x^3 + 3x^2 - 9x + 4$$

Example 3. Increasing and Decreasing

Find the critical numbers and decide where f is increasing and decreasing if $f(x) = (x-1)^{3/2}$

Example 4. Increasing and Decreasing

Find the intervals for which the following function increases and decreases. Graph the function.

$$f(x) = \frac{x-1}{x+1}$$

Example 5. Profit Analysis

A company selling computers finds that the cost per computer decreases linearly with the number sold monthly, decreasing from \$1000 when none are sold to \$800 when 1000 are sold. Suppose the revenue function can be approximated by $R(x) = 0.0008x^3 - 2.4x^2 + 2400x$. Determine any intervals on which the profit function is increasing.

Example 1. Maximizing Viewer's Attention

Suppose that the manufacturer of a diet soft drink is disappointed by sales after airing a new series of 30-second television commercials. The company's market research analysts hypothesize that the problem lies in the timing of the commercial's message. Either it comes too early in the commercial, before the viewer has become involved; or it comes too late, after the viewer's attention has faded. After extensive experimentation, the research group finds that the percent of full attention that a viewer devotes to a commercial is a function of time since the commercial began where

$$f(t) = -\frac{3}{20}t^2 + 6t + 20 \quad 0 \leq t \leq 30$$

When is the best time to present the commercial's sales

Example 2. Relative Extrema

Identify the x-values of all points where the graph has relative extrema

Example 3. Relative Extrema

Find all relative exrema for the following functions

(a) $f(x) = 2x^3 - 3x^2 - 72x + 15$

(b) $f(x) = 6x^{3/2} - 4x$

(c) $f(x) = x^3 e^x$

Example 4. Bicycle Sales

A small company manufactures and sells bicycle. The production manager has determined that the cost and demand fucntions for q bicycles per week are

$$C(q) = 10 + 5q + \frac{1}{60}q^3$$

and

$$p = D(q) = 90q - q^2$$

(a) Find the maximum weekly revenue.

(b) Fnd the maximum weekly profit.

(c) Find the price the company should charge to realize maximum profit.