Math 116: Business Calculus
Chapter 4 - Calculating Derivatives

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Exam 2 - Thursday March 9.

4.1 Techniques for Finding Derivatives.

4.2 Derivatives of Products and Quotients.

4.3 The Chain Rule.

4.4 Derivatives of Exponential Functions.

4.5 Derivatives of Logarithmic Functions.

5.1 Increasing and Decreasing Functions.

5.2 Relative Extrema.

Notation: Derivatives

- $f'(x)$
- $\dfrac{dy}{dx}$
- $\dfrac{d}{dx}[f(x)]$
- $D_x[f(x)]$

Example 1. Derivative of a Constant

If $f(x) = k$, where k is any real number, then

$$f'(x) = \frac{d[k]}{dx} = 0$$

Use the definition of a derivative to show this!

(a) If $f(x) = 9$, then $f'(x) =$

(b) If $h(t) = 9$, then $D_t[h(t)] =$

(c) If $y = 2^3$, then $dy/dx =$

Example 2. Power Rule

If $f(x) = x^n$, where n is any real number, then

$$f'(x) = \frac{d[x^n]}{dx} = n\,x^{n-1}$$

Use the definition of a derivative to show this!

(a) If $f(x) = x^6$, then $f'(x) =$

(b) If $y = t$, then $dy/dx =$

(c) If $y = 1/x^3$, then $dy/dx =$

(d) $D_x[x^{4/3}] =$

(e) $D_z[\sqrt{z}] =$

Example 3. Derivative of a Constant Times a Functions

Let k be a real number. If $g'(x)$ exists, then the derivative of $f(x) = k \cdot g(x)$ is

$$f'(x) = \frac{d}{dx}[k \cdot g(x)] = k \cdot g'(x)$$

Use the definition of a derivative to show this!

(a) If $y = 8x^4$, fnd dy/dx

(b) If $y = -\frac{3}{4}x^{12}$, find dy/dx

(c) Find $D_t(-8t)$

(d) Find $D_p[10p^{3/2}]$

(e) If $y = 6/x$, find dy/dx

Example 5. Derivative of a Sum

If $f(x) = u(x) \pm v(x)$ and if $u'(x)$ and $v'(x)$ exist, then

$$f'(x) = \frac{d}{dx}[u(x) \pm v(x)] = u'(x) \pm v'(x)$$

Find the derivative of each function.

1. $y = 6x^3 + 15x^2$
2. $p(t) = 12t^4 - 6\sqrt{t} + \frac{5}{t}$
3. $f(x)\dfrac{x^3 + 3\sqrt{x}}{x}$
4. $g(x) = (4x^2 - -3x)^2$

Example 6. Marginal Cost

Suppose that the total cost in hundreds of dollars to produce x thousand barrels of a beverage is given by

$$C(x) = 4x^2 + 100x + 500$$

Find the marginal cost for the following values of x

(a) $x = 5$

(b) $x = 30$

Example 7. Marginal Revenue

The demand function (in dollars) for a certain product is given by

$$p = \frac{50,000 - q}{25,000}$$

for $0 \leq q \leq 50,000$. Find the marginal revenue when $q = 10,000$ units.

Example 8. Marginal Revenue

Economists usually give demand functions with the quantity as the independent variable, as in Example 7. It might make more intuitive sense, however, to think of the quantity demanded as a function of the price. For example, suppose the demand function is given as

$$q = 50,000 - 25,000p$$

where p is the price in dollars and $0 \leq p \leq 2$. Find the marginal revenu when the price is \$1.60

Example 9. Marginal Profit

Suppose that the cost function for the product in Example 7. is give by

$$C(q) = 2,100 + .25q$$

where $0 \leq q \leq 50,000$ Find the marginal profit from the production of the the following number of units.

(a) 15,000

(b) 21,875

(c) 25,000

Homework 4.1 - Problem 56

An analyst has found that a company's costs and revenues in dollars for one product are given by

$$C(x) = 2x \quad R(x) = 6x = \frac{x^2}{1000}$$

(a) Find the marginal cost function.

(b) Find the marginal revenue function.

(c) Find the marginal profit function.

(d) What value of x makes marginal profit equal to 0?

(e) Find the profit when the marginal profit is 0.

Example 1. Product Rule

If $f(x) = u(x) \cdot v(x)$ and if $u'(x)$ and $v'(x)$ both exist, then

$$f'(x) = \frac{d}{dx}[u(x) \cdot v(x)] = u(x) \cdot v'(x) + v(x) \cdot u'(x)$$

Let $f(x) = (2x+3)(3x^2)$. Use the product rule to find $f'(x)$.

Example 2. Product Rule

Find the derivative of $y = (\sqrt{x} + 3)(x^2 - 5x)$

Example 3. Quotient Rule

If $f(x) = u(x)/v(x)$ and if $u'(x)$ and $v'(x)$ both exist, and if $v(x) = 0$then

$$f'(x) = \frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{v(x)\cdot u'(x) - u(x)\cdot v'(x)}{[v(x)]^2}$$

Find $f'(x)$ if $f(x) = \dfrac{2x-1}{4x+3}$

Example 4. Product and Quotients

Find $D_x \left[\dfrac{(3-4x)(5x+1)}{(7x-9} \right]$

Minimum Average Cost

Suppose the cost in dollars of manufacturing x hundred small motors is given by

$$C(x) = \frac{3x^2 + 120}{2x + 1}$$

(a) Find the average cost per hundred motors.

(b) Find the marginal average cost.

(c) As we shall see in the next chapter, average cost is generally minimized when the marginal average cost is zero. Find the level of production that minimizes the average cost.

Example 1. Composite Functions

Let $f(x) = 2x - 1$ and $g(x) = \sqrt{3x + 5}$

(a) $g[f(4)]$

(b) $f[g(4)]$

(c) $f[g(-2)]$

Example 2. Composition of Functions

Let $f(x) = 2x^2 + 5x$ and $g(x) = 4x + 1$. Find the following

(a) $f[g(x)]$

(b) $g[f(x)]$

Example 3. Composition of Functions

Write each function as the composition of two functions f and g so that $h(x) = f[g(x)]$.

(a) $h(x) = 2(4x+1)^2 + 5(4x+1)$

(b) $h(x) = \sqrt{1-x^2}$

Example 5. Chain Rule

Find dy/dx if $y = (3x^2 - 5x)^{1/2}$.

Example 6. Chain Rule

Use the chain rule to find $D_x(x^2+5x)^8$.

Example 7. Derivative Rules

Find the derivative of $y = 4x(3x+5)^5$.

Example 8. Derivative Rules

Find $D_x\left[\dfrac{(3x+2)^7}{x-1}\right]$.

Example 10. Compound Interest

Suppose a sum of \$500 is deposited in an account with an interest rate of r percent per year compounded monthly. At the end of 10 years, the balance in the account is given by

$$A = 500\left(1 + \frac{r}{1200}\right)^{120}$$

Find the rate of change of A with respect to r if $r = 5$.

Problems

Find $\frac{dy}{dx}$ if

21. $y = 8x^4 - 5x^2 + 1)^4$

27. $y = -3\sqrt{7t^3 - 1}$

Problems

Find $\frac{dy}{dx}$ if

21. $y = 8x^4 - 5x^2 + 1)^4$

$$\frac{dy}{dx} = 4(8x^4 - 5x^2 + 1)^3(32x^3 - 10x)$$

27. $y = -3\sqrt{7t^3 - 1}$

$$\frac{dy}{dx} = \frac{-63t^2}{2\sqrt{7t^3 - 1}}$$

Demand

Suppose a demand function is given by

$$q = D(p) = 30\left(5 - \frac{p}{\sqrt{p^2+1}}\right)$$

where q is the demand for a product and p is the price per item in dollars. Find the rate of change in the demand for the product per unit change in price.

Derivatives of exponentials

Example 1.

Find the derivative of each function

(a) $y = e^{5x}$

(b) $s = 3^t$

(c) $y = 10e^{x^2}$

(d) $s = 8 \cdot 10^{1/t}$

Example 2.

Let $y = e^{x^2+1}\sqrt{5x+2}$. Find dy/dx.

Example 3.

Let $f(x) = \dfrac{100,000}{1+100e^{-0.3x}}$. Find $f'(x)$.

Example 5. Product Sales

A company sells 900 units of a new product in the first year and 3213 units in the fourth year. They expect that sales acan be approximated by a logistic functin, leveling off at around 100,000 in the long run.

(a) Find a formula $S(t)$ for the sales as a function of time.

(b) Find the rate of change of sales after 4 years.

Example 1. Derivatives of Logarithmic Functions

Find the derivate of each function

(a) $f(x) = \ln(6x)$

(b) $y = \log(x)$

Example 2. Derivatives of Logarithmic Functions

Find the derivate of each function

(a) $f(x) = \ln(x^2 + 1)$

(b) $y = \log_2(3x^2 - 4x)$

Example 3. Derivatives of Logarithmic Functions

Find the derivate of each function

(a) $f(x) = \ln(|5x|)$

(b) $f(x) = 3x \cdot \ln(x^2)$

Example 4. Automobile Resale Value

Based on projections from the Kelley Blue Book, the resale value of a 2014 Toyota 4Runner SR5 can be approximated by the following function

$$f(t) = 30,781 - 24,277 \log(0.46t + 1)$$

where t is the number of years since 2014. Find and interpret $f(4)$ and $f'(4)$.