Math 116: Business Calculus
Chapter 3 - Nonlinear Functions

Instructor: Colin Clark

Spring 2017

Exam 1 - Thursday February 9.

1.1 Slopes and Equations of Lines.

1.2 Linear Functions and Applications.

2.1 Properties of Functions.

2.2 Quadratic Functions.

2.3 Polynomial and Rational Functions.

2.4 Exponential Functions.

2.5 Logarithmic Functions.

2.6 Applications

3.1 Limits.

3.3 Rates of Change.

3.4 Definition of the Derivative.

Example 1. Finding a limit

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$f(x)$	3.61	3.9601	3.99960001	4	4.00040001	4.004001	4.0401	4.41

Notation

1. $\lim_{x \to a^+} f(x)$
2. $\lim_{x \to a^-} f(x)$

Definition

Limit of a function

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Limit of a function:
Let $f(x)$ be a function and let a and L be real numbers. If

1. 1. as x takes values closer and closer (but not equal) to a on both sides of a, the corresponding values of $f(x)$ get closer and closer to L
2. the value of $f(x)$ can be made as close to L as desired by taking values of x close enough to a

then L is the limit of $f(x)$ as x approaches a, written

$$\lim_{x \to a} f(x) = L$$

Example 2. Finding a limit

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Example 3. Finding a limit

Determine $\lim_{x \to 2} h(x)$ for the function h defined by:

$$h(x) = \begin{cases} x^2 & x \neq 2 \\ 1 & x = 2 \end{cases}$$

Example 4. Finding a limit

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Construct a table with x values getting close and closer to 2 and find the corresponding values of $g(x)$.

x	-2.1	-2.01	-2.001	-2.0001	2	-1.9999	-1.999	-1.99	-1
$f(x)$									

Example 5. Finding a limit

Find $\lim_{x \to 0} \frac{\|x\|}{x}$

Existence of Limits

Example 6. Finding a limit

Suppose $\lim_{x\to 2} f(x) = 3$ and $\lim_{x\to 2} g(x) = 4$. Usethe limit rules to find the following limits:

a. $\lim_{x\to 2}\left[f(x)+5g(x)\right]$

b. $\lim_{x\to 2}\dfrac{\left[f(x)\right]^2}{\ln(g(x))}$

Example 7. Finding a limit

Find $\lim_{x \to 3} \dfrac{x^2 - x - 1}{\sqrt{x+1}}$

Example 8. Finding a limit

Find $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

Example 9. Finding a limit

Find $\lim_{x \to 4} \dfrac{\sqrt{x} - 2}{x - 4}$

Example 10. Finding a limit

Find $\lim_{x \to 1} \dfrac{x^2 - 2x + 1}{(x-1)^3}$

Example 12. Finding a limit

Find

a. $\lim_{x\to\infty} \dfrac{8x+6}{3x-1}$

b. $\lim_{x\to\infty} \dfrac{3x+2}{4x^3-1}$

Example 12. Finding a limit

Find

c. $\lim_{x\to\infty} \dfrac{3x^2+2}{4x-1}$

d. $\lim_{x\to\infty} \dfrac{5x^2-4x^3}{3x^2+2x-1}$

Review of Terminology

- Limit
- Two sided limit
- Limit from the left / limit from the right
- existence of limits

Rates of Change

1. Average rate of change / Secant Line
2. Instantaneous rate of change / Tangent line

Example 2. Education Spending

The figure show the total amount appropriated annually (in billions of dollars) for the U.S. Department of Education in recent years. Find the average rate of change per year in Department of Education appropriation from 2009 to 2013

Example 4. Manufacturing

A company determines that the cost in dollars to manufacture x gallons of perfume is given by

$$C(x) = 100 + 15x - x^2 \quad 0 \leq x \leq 7$$

a. Find the average rate of change, per gallon, in the total cost if the nubmer of gallons manufactured changes from 1 to 5 cases.

b. Find the additional cost when production is increased from 1 to 2 gallons

Example 5. Manufacturing

A company determines that the cost in dollars to manufacture x gallons of perfume is given by

$$C(x) = 100 + 15x - x^2 \quad 0 \leq x \leq 7$$

c. Find the instantaneous rate of change of cost with respect to the number of gallons produced when just one gallon is produced.

d. ... when 5 gallons are produced

Review of Terminology

- Average rate of change
- Instantaneous rate of change
- Marginal cost

Example 1. Tangent Line

Consider the graph of $f(x) = x^2 + 2$

a. Find the slope and equation of the secant line through the points where $x = -1$ and $x = 2$

b. Find the slope and equation of the tangent line at $x = -1$

Example 2. Slope

Use a graphing calculator to find the slope of $f(x) = x^x$ at $x = 1$

Example 4. Derivative

Let $f(x) = x^2$

a. Find the derivative.

b. Calculate and interpret $f'(-3)$

Example 5. Derivative

Let $f(x) = 2x^3 + 4x$. Find

a. $f'(x)$

b. $f'(2)$

c. $f'(-3)$

Example 6. Derivative

Let $f(x) = \dfrac{4}{x}$ Find $f'(x)$

Example 8. Cost Analysis

The cost in dollars to manufacture x graphing calculators is given by $C(x) = -0.005x^2 + 20x + 150$ when $0 \leq x \leq 2000$ Find the rate of change of cost with respect to the number manufactured when 100 calculators are made and whn 1000 calculators are made.

Example 9. Tangent Line

Find the equation of the tangent line to the graph of $f(x) = 4/x$ at $x = 2$.

Review of Terminology

- Derivative
- Tangent Line
- slope of the curve at a point
- differentiation
- Differentiable at x
- Difference Quotient