Math 116: Business Calculus
Chapter 2 - Nonlinear Functions

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Spring 2017

Exam 1 - Thursday February 9.

1.1 Slopes and Equations of Lines.

1.2 Linear Functions and Applications.

2.1 Properties of Functions.

2.2 Quadratic Functions.

2.3 Polynomial and Rational Functions.

2.4 Exponential Functions.

2.5 Logarithmic Functions.

2.6 Applications

3.1 Limits.

3.3 Rates of Change.

3.4 Definition of the Derivative.

Example 1. Dow Jones

An important function to investors around the world is the Dow Jones industrial average, a performance measure of the stock market.

a. Is $f(t)$ a linear function.

b. Determine the domain and range of the function.

c. Estimate $f(14)$.

d. Solve the equation $f(t) = 14.250$

Example 4. Domain and Range

Find the domain and range of the functions defined below:

1. $f(x) = x^2$

2. $y = x^2$ with the domain specified as $\{-2, -1, 0, 1, 2\}$.

3. $y = \sqrt{6 - x}$

4. $y = \sqrt{2x^2 + 5x - 12}$

Example 4. Domain and Range

Find the domain and range of the functions defined below:

1. $f(x) = x^2$
   Domain: $(-\infty, \infty)$. Range: $[0, \infty)$
2. $y = x^2$ with the domain specified as $\{-2, -1, 0, 1, 2\}$.
   Domain: $-2, -10, 1, 2$. Range: $0, 1, 4$
3. $y = \sqrt{6 - x}$
   Domain: $(-\infty, 6]$. Range: $[0, \infty)$
4. $y = \sqrt{2x^2 + 5x - 12}$
   Domain: $(-\infty, -4] \cup [3/2, \infty)$. Range: $[0, \infty)$

Example 5. Evaluating Functions

Let $g(x) = -x^2 + 4x - 5$. Evaluate:

a. $g(3)$

b. $g(a)$

c. $g(x+h)$

d. $g(2/r)$

e. Find all values of x such that $g(x) = -12$

Example 5. Evaluating Functions

Let $g(x) = -x^2 + 4x - 5$. Evaluate:

a. $g(3) = -2$

b. $g(a) = -a^2 + 4 - 5$

c. $g(x+h) = -x^2 - 2xh - h^2 + 4x + 4h - 5$

d. $g(2/r) = -4/r^2 + 8/r - 5$

e. Find all values of x such that $g(x) = -12$
$\Rightarrow x = 2 \pm \sqrt{11} \approx -1.317, 5.317$

Problem 56. Evaluating Functions

Let $f(x) = -\dfrac{1}{x^2 - 2x}$. Evaluate:

a. $f(4)$

b. $f(a)$

c. $f(x+h)$

Example 6. Vertical Line Test

State the vertical line test.

Example 7. Even and Odd Functions

Determine whether each of the following functions is even, odd, or neither.

a. $f(x) = x^4 - x^2$

b. $f(x) = \dfrac{x}{x^2+1}$

c. $f(x) = x^4 - 4x^3$

Example 8. Delivery Charges

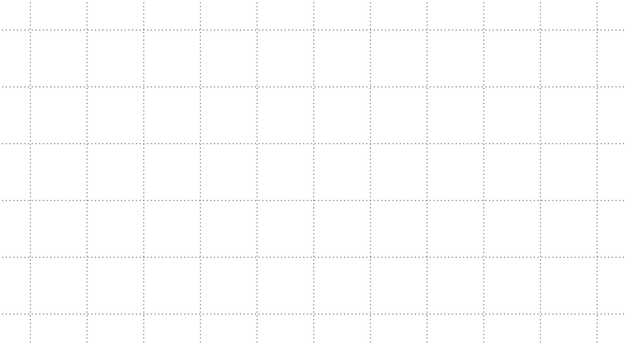
An overnight delivery service charges \$25 for a package weighing up to 2 lb. For each additional pound, or portion thereof, there is an additional charge of \$3. Let $D(w)$ represent the cost to send a package weighing w lb. Graph $D(w)$ for w in the interval $(0, 6]$

Review of Terminology

- A **function** is a rule that assigns to each element from one set exactly one element from another set.
- Independent variable. Dependent variable.
- The set of all possible values of the independent variable in a function is called the **domain** of the function.
- The resulting set of possible values of the dependent variable is called the **range** of the function.

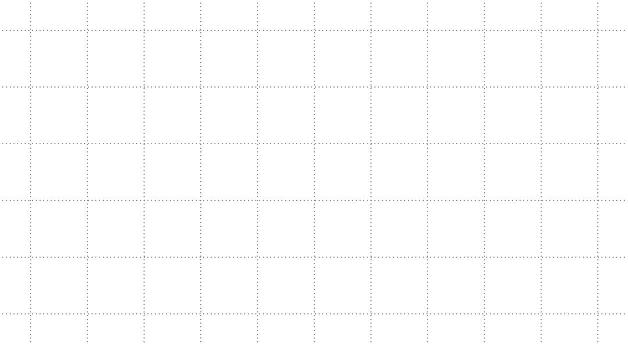
Example 1. Graphing Quadratics

Graph $y = x^2 - 4$



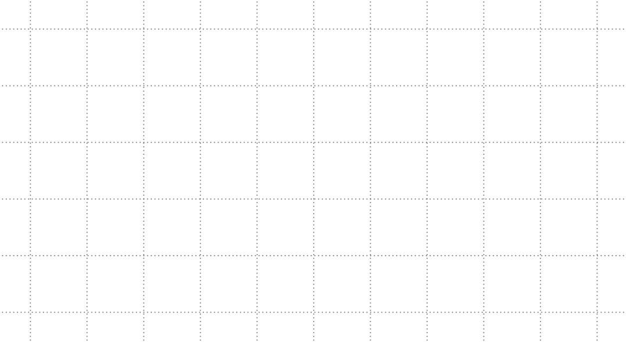
Example 2. Graphing Quadratics

Graph $y = ax^2$ for $a = -0.5, a = -1, a = -2$



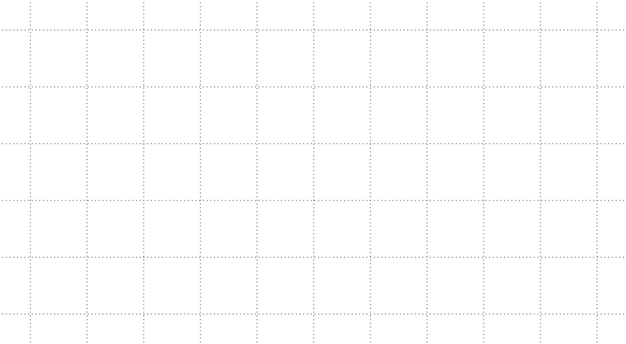
Example 5. Graphing Quadratics

Graph $f(x) = -4x^2 - 8x + 5$

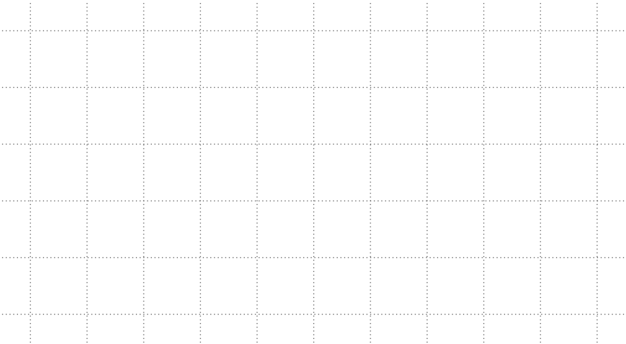


Example 6. Graphing Quadratics

Graph $y = x^2 + 4x + 6$



Example 6. Graphing Quadratics



Example 7. Management Science

When Power and Money, Inc. charges \$600 for a seminar on management techniques, it attracts 100 people. For each \$20 decrease in the fee, an additional 100 people will attend the seminar. The managers are wondering how much to charge for the seminar to maximize their revenue.

Example 8. Profit

A deli owner has found that his revenue from producing x pounds of cream cheese is given by $R(x) = -x^2 + 30x$, while the cost in dollars is given by $C(x) = 5x + 100$.

a. Find the minimum break-even quantity.

b. Find the maximum revenue.

c. Find the maximum profit.

Example 8. Profit

A deli owner has found that his revenue from producing x pounds of cream cheese is given by $R(x) = -x^2 + 30x$, while the cost in dollars is given by $C(x) = 5x + 100$.

a. Find the minimum break-even quantity.
   $x = 5$

b. Find the maximum revenue.
   \$225

c. Find the maximum profit.
   Selling 12.5lbs produces a max profit of \$56.25.

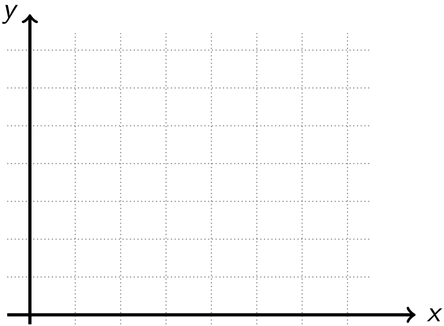
Review of Terminology

What do each of the following refer to:

- Quadratic Function
- Parabola
- Zeros
- Vertex

Example 1. Translations and Reflections

Graph $f_1(x) = x^3$ and $f_2(x) = -(x-2)^3 + 3$ below.



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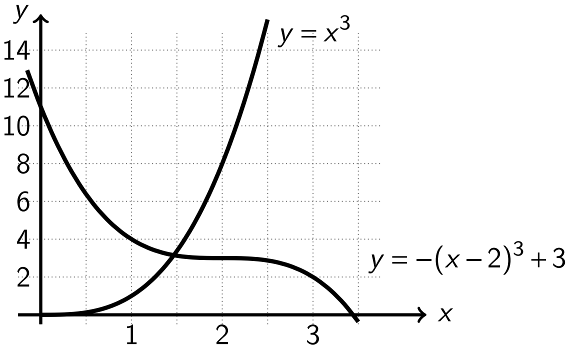

Example 2. Graphing a Polynomial

Use a graphing calculator to find a suitable window to graph $g(x) = 8x^3 - 12x^2 + 2x + 1$ below.

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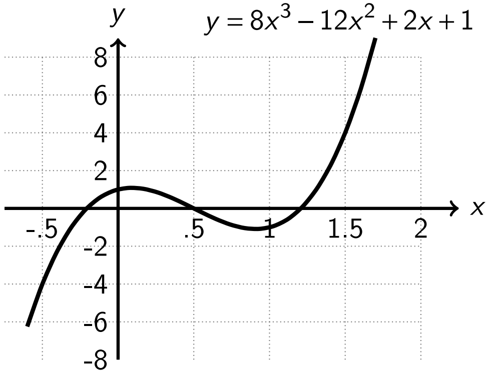

Example 3. Graphing a Polynomial

Use a graphing calculator to find a suitable window to graph $h(x) = -3x^4 + 14x^3 - 54x + 3$ below.

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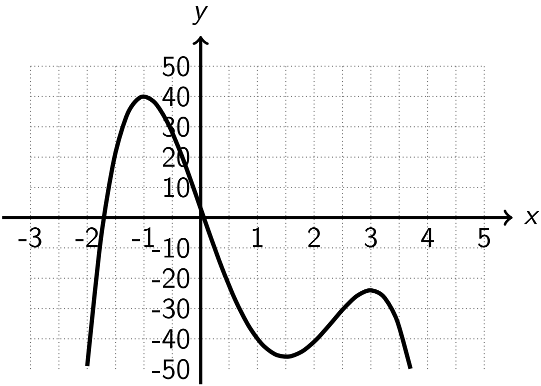
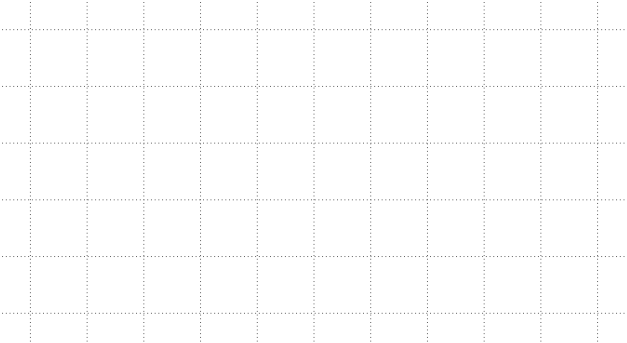


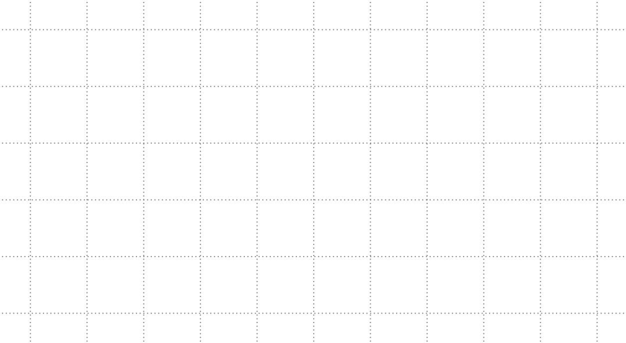
Example 4. Degree of a Polynomial

Identify the degree of the polynomial in each of the figures, and give the sign for the leading coefficient.



Example 6. Graphing Rational Functions

Graph $y = \dfrac{1}{x}$.



Example 7. Graphing Rational Functions

Graph $y = \dfrac{x^2+3x+2}{x+1}$

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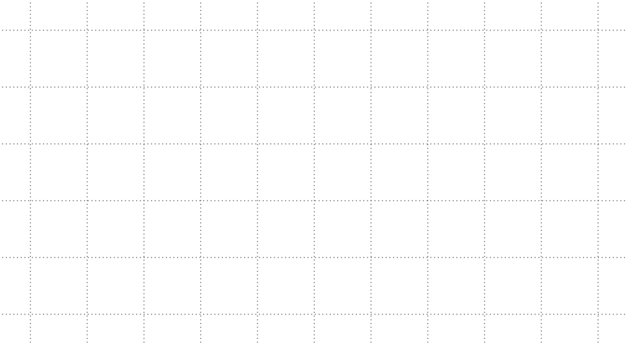


Example 8. Cost-Benefit Analysis

Suppose a cost-benefit model where y is the cost (in thousands of dollars) of removing x percent of a certain pollutant is given by

$$y = \frac{18x}{106 - x}.$$

1. What is the domain of x?
2. Find the cost to remove 100%, 95%, 90% and 80%.
3. Graph the function.

Example 8. Cost-Benefit Analysis

Suppose a cost-benefit model where y is the cost (in thousands of dollars) of removing x percent of a certain pollutant is given by

$$y = \frac{18x}{106 - x}.$$

1. What is the domain of x?
   $x \leq x \leq 100$
2. Find the cost to remove 100%, 95%, 90% and 80%.
   300, 155, 101, 55.
3. Graph the function.

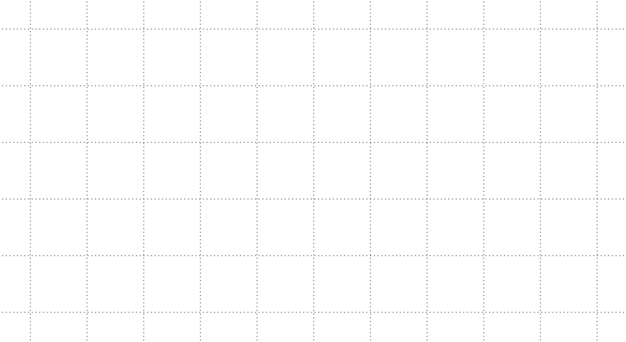
Review of Terminology

What do each of the following refer to:

- Polynomial Function
- Rational Function
- Degree, Leading coefficient
- Vertical and Horizontal Asymptote
- Average Cost

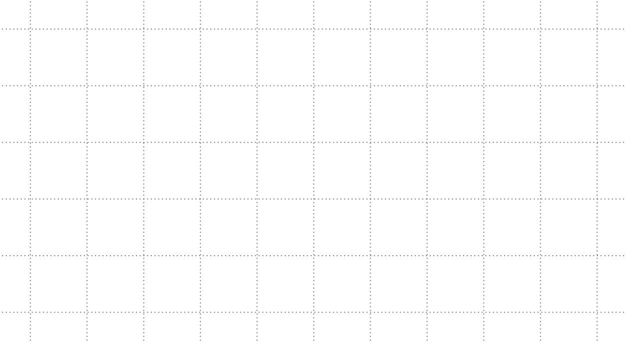
Example 1. Graphing Exponentials

Graph $f_1(x) = 2^{-x}$



Example 2. Graphing Exponentials

Graph $f_2(x) = -2^x + 3$



Solving Exponentials Equations

1. Solve $9^x = 27$
2. Solve $32^{2x-1} = 128^{x+3}$

Solving Exponential Equations

1. Solve $9^x = 27 \quad \Rightarrow \quad x = 3/2$
2. Solve $32^{2x-1} = 128^{x+3} \quad \Rightarrow \quad x = 26/3$

Interest

1. Simple interest

2. Compounding Periodically

3. Compounding Continuously

Example 4. Compound Interest

Morgan Presley invests a bonus of \$9,000 at 6% annual interest compounded semiannually for 4 years. How much interest will she earn?

Continuous Compound Interest

If \$600 is invested in an account earning 2.75% compounded continuously, how much would be in the account after 5 years?

Problem 40

Find the interest rate required for an investemnt of $5000 to grow to $6100 in 5 years if interest is compounded as follows:

1. Annually
2. Quarterly
3. Continously (Requires Section 2.5)

Homework 2.4 - Problem 44

Lauren Snowden puts $10,500 into an account to save money to buy a car in 12 years. She expects the car of her dreams to cost $35,000 by then. Find the interest rate that is necessary if the interest is computed using the following methods.

a. Compounding quarterly.

b. Compounding monthly.

c. Compounding continously.

Review of Terminology

State the meaning of:

- Exponential Function
- Simple Interest
- Principal, Return, Rate of Interest, compound amount
- Compound Interest
- Continuous Compounding

Example 4. Evaluating Logarithms

Evaluate $log_5(80)$

Example 6. Solving Exponential Equations

Solve each equation.

1. $3^x = 5$
2. $3^2x = 4^{x+1}$
3. $5e^{0.01x} = 9$

Change-of-Base Theorem

1. Write 7^x using base e rather than base 7.
2. Approximate the function $f(x) = e^{2x}$ as $f(x) = a^x$ for some base a.

Example 8. Doubling Time

With an inflation rate averaging 5% per year, how long will it take for prices to double?

Review of Terminology

- Logarithmic Function
- Common Logarithm
- Natural Logarithm
- Double Time
- Base, Change of Base.
- $a^x = e^{(\ln a)x}$
- $\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

Example 4. Effective Rate

Find the effective rate corresponding to each stated rate.

a. 6% compounded quarterly

Example 4. Effective Rate

Find the effective rate corresponding to each stated rate.

b. 6% compounded continuously

Example 5. Time of Investment

Susan Freilich has received a bonus of $25,000. She invests it in an account earning .2% compounded quarterly. Find how long it will take for her investment to grow to $40,000.

Example 6. Present Value

Tom Shaffer has a balloon payment of \$100,000 due in 5 years. What is the present value of that amount if the money earns interest at 4.5% annually?

Example 7. Continuous Compound Interest Rate

Find the interest rate that will cause $5,000 to grow to $3,700 in 6 years if the money is compounded continuously.

Example 8. Employee Turnover

Assembly-line operations tend to have a high turnover of employees, forcing companies to spend much time and effort in training new workers. It has been found that a worker new to a task will produce items according to the function defined by

$$P(x) = 25 - 25e^{-0.3x}$$

where $P(x)$ items are produce by the worker on day x.

a. What happens to the number of items a worker can produce as x gets larger and larger?

b. How many days will it take for a new worker to produce at least 20 items in a day?

Review of Terminology

- Effective rate for compound interest
- Nominal rate
- Present value
- Limited growth function