Math 116: Business Calculus

Instructor: Colin Clark

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Exam 1 - Thursday February 9.

1.1 Slopes and Equations of Lines.

1.2 Linear Functions and Applications.

2.1 Properties of Functions.

2.2 Quadratic Functions.

2.3 Polynomial and Rational Functions.

2.4 Exponential Functions.

2.5 Logarithmic Functions.

2.6 Applications

3.1 Limits.

3.3 Rates of Change.

3.4 Definition of the Derivative.

Slope of a Line

$$\text{slope} = m = \frac{\text{Change in } y}{\text{Change in } y} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of a Line

Find the slope of the line through each pair of points.

a. $(7, 6)$ and $(-4, 5)$

b. $(5, -3)$ and $(-2, -3)$

c. $(2, -4)$ and $(2, 3)$

Equation of a Line

An equation in two first-degree variables is called a linear equation and has a straight line as its graph. Which of the following equations are linear?

- $4x + 7y = 20$
- $3x^2 - y^2 + 2 = 0$
- $x + 2x + 3x = y + 6$
- $x + 2x + 3x = y + 6x$
- $xy = 8$
- $y = 5x - 2$

Horizontal and Vertical Lines

1. Horizontal Line:
   The slope of a horizontal line is 0.
2. Vertical Line:
   The slope of a vertical line is undefined.

Slope-Intercept Form

If a line has slope m y-intercept b, then the equation of the line in slope intercept form is

$$y = mx + b$$

Point-Slope Form

If a line has slope m and passes through the point x_1, y_1, then a point-slope equation of the line is

$$y - y_1 = m(x - x_1)$$

Point-Slope Form with Two Points

Either

1. First find the slope, then find the equation of the line using point slope form.
   OR
2. Find the the equation of the line using point-point form and then simplify the algebra

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Parallel and Perpendicular Lines

1. Two lines are parallel if and only if they have the same slope, or if they are both vertical.
2. Two lines are perpendicular if and only if the product of their slopes is -1, or if one of the lines is vertical and the other line is horizontal.

Problem 65 - Consumer Price Index

The consumer Price Index (CPI) is a measure of the change in the cost of goods over time. The index was 100 for the three-year period centered on 1983. For simplicity, we will assume that the CPI was exactly 100 in 1983. Then the CPI of 229.6 in 2012 indicates that an item that cost \$1.00 in 1983 would cost \$2.30 in 2012. The CPI has been increasing approximately linearly over the last few decades.

a. Use this information to determine an equation for the CPI in terms of t, which represents the years since 1980.

b. Based on the answer to (a), what was the predicted value of the CPI in 2000? Compare this estimate with the actual CPI of 172.2.

c. Describe the rate at which the annual CPI is changing.

Problem 65 - Solutions

a. Use this information to determine an equation for the CPI in terms of t, which represents the years since 1980.
$y = 4.469t + 86.593$

b. Based on the answer to (a), what was the predicted value of the CPI in 2000? Compare this estimate with the actual CPI of 172.2.
Predicted CPI is 176 which is slightly higher than actual CPI.

c. Describe the rate at which the annual CPI is changing.
CPI is increasing at a rate of about 4.5 units per year.

Problem 64 - Use of Cellular Telephones

The following table shows the subscribership of cellular telephones in the United States (in millions) for selected years between 2000 and 2012.

Year	2000	2004	2008	2012
Subscribers	109.48	182.14	270.33	326.48

a. Accurately plot the data by letting $t = 0$ correspond to 2000. Discuss how well the data fit a straight line.

b. Determine a linear equation that approximates the number of subscribers using the points $(0, 109.48)$ and $(12, 326.48)$.

Problem 64 - Use of Cellular Telephones

The following table shows the subscribership of cellular telephones in the United States (in millions) for selected years between 2000 and 2012.

Year	2000	2004	2008	2012
Subscribers	109.48	182.14	270.33	326.48

c. Repeat (b) using the points $(4, 182.14)$ and $(12, 326.48)$.

d. Discuss why your answers to parts (b) and (c) are similar, but not identical.

e. Using your equations from (b) and (c), approximate the number of cellular phone subscribers in the year 2010. Compare your results with the actual value of 296.29 million.

Review of Terminology

Write one sentence explaining what is meant by each of the following terms.

1. Ordered pairs, x-coordinate, y-coordinate
2. Cartesian coordinate system, x-axis, y-axis, origin
3. Graph
4. x-intercept, y-intercept
5. Slope of a line
6. Linear equation
7. Slope-intercept form
8. Point-slope form

Function Notation

Letters such as f, g or h are often used to name functions. For example, f might be used to name the function defined by

$$y = 5 - 3x$$

by replacing y with $f(x)$ to get

$$f(x) = 5 - 3x$$

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Linear Functions

A linear equation, in the form $y = mx + b$ may be used to define a function f by

$$f(x) = mx + b$$

Supply and Demand

1. The law of demand states that if all other factors remain equal, as the price of an item increases, consumers are less likely to buy an increasingly expensive item and so the demand for the item decreases.
2. The law of supply states that as the price of the item increases, producers are more likely to see a profit in selling the item and will increase the production and the supply of the item increases.

Example 2

Suppose that Greg Tobin, manager of a giant supermarket chain has studied the supply and demand for watermelons.

a. He has noticed that the demand increases as the price decreases. He has determined that the quantity (in thousands) demanded weekly, q and the price, in dollars per watermelon, p, are related by the linear function

$$p = D(q) = 9 - 0.75q$$

Find the quantity demanded at a price of \$5.25 per watermelon and at a price of \$3.75 per watermelon.

Example 2

Suppose that Greg Tobin, manager of a giant supermarket chain has studied the supply and demand for watermelons.

b. Greg also noticed that the quantity of watermelons supplied decreased as the price decreased. Price, p, and supply, q, are related by the linear function

$$p = S(q) = 0.75q$$

Find the quantity supplied at a price of \$5.25 per watermelon and at a price of \$3.00 per watermelon.

Example 2

Suppose that Greg Tobin, manager of a giant supermarket chain has studied the supply and demand for watermelons.

c. Graph both functions on the same axes.

Equilibrium Quantity

When supply and demand are equal, the economy is said to be in equilibrium. The equilibrium price of the commodity is the price found at the point where the supply and demand graphs for that commodity intersect. The equilibrium quantity is the quantity demanded and supplied at the same point.

Example 3

Suppose that Greg Tobin, manager of a giant supermarket chain has studied the supply and demand for watermelons.

d. Use algebra to find the equilibrium quantity and price for the watermelons.

Cost Analysis

When the cost to manufacture an item consists of **fixed costs**, (associated with designing the product, setting up a factory, training workers, etc) and **cost per item** (labor, materials, packaging and shipping etc.), then the cost function is linear and $C(x) = mx + b$.

Example 5

The marginal cost to make x batches of a prescription medication is \$10 per batch, while the cost to produce 100 patches is \$1500. Find the cost function $C(x)$, given that it is linear.

Example 5 - Solution

The marginal cost to make x batches of a prescription medication is \$10 per batch, while the cost to produce 100 patches is \$1500. Find the cost function $C(x)$, given that it is linear.

$C(x) = 10x + 500$

Example 6 - Cost Function

A company has fixed costs of \$12,500. The total cost to produce 1000 widgets is \$13,260. Find the cost function $C(x)$, given that it is linear.

Break-Even Analysis

Let p be the price per unit and let x be the number of units sold.

The **revenue** $R(x)$ is the amount of money that a company receives. It is the product of the number of units sold and the price per unit.

The **profit**, $P(x)$, is the money a company makes after paying its costs. It is the difference between revenue and costs.

The **break-even quantity** is found by setting profit equal to zero.

The **break-even point** is the corresponding ordered pair and indicates the quantity and price where there is zero profit or loss.

Example 7 - Break Even Analysis

A firm producing poultry feed finds that the total cost $C(x)$, in dollars of producing and selling x units is given by $C(x) = 20x + 100$. Management plans to charge \$24 per unit for the feed.

a. Determine the revenue function.

b. How many units must be sold for the firm to break even?

c. What is the profit if 100 units of feet are sold?

d. How many units of feet must be sold to produce a profit of \$900.

Example 7 - Solutions

A firm producing poultry feed finds that the total cost $C(x)$, in dollars of producing and selling x units is given by $C(x) = 20x + 100$. Management plans to charge $24 per unit for the feed.

a. Determine the revenue function. $R(x) = 24x$

b. How many units must be sold for the firm to break even? $x = 25$

c. What is the profit if 100 units of feet are sold? $300

d. How many units of feet must be sold to produce a profit of $900. 250 units

Problem 28 - Homework

Suppose that the demand and price for strawberries are related by $p = D(q) = 5 - 0.25q$, where p is the price (in dollars) and q is the quantity demanded (in hundreds of quarts)

a. Find the price at each level of demand.
   0 quarts, 400 quarts, 840 quarts.
b. Find the quantity demanded for the strawberries at each price.
   \$4.50, \$3.25, \$2.40.
c. Graph the demand function.

Problem 28 - Homework

Suppose the price and the supply of strawberries are related by $p = S(q) = 0.25q$, where$p$ is the price (in dollars) and q is the quantity demanded (in hundreds of quarts).

d. Find the quantity supplied at each price.
   \$0.00, \$2.00, \$4.50.

e. Graph the supply function the same figure as before.

f. Find the equilibrium quantity and the equilibrium price.

Review of Terminology

Write one sentence explaining what is meant by each of the following terms.

1. linear function, linear cost function
2. independent/dependent variable
3. supply and demand curves
4. surplus, shortage
5. equilibrium quantity, equilibrium price, equilibrium point
6. fixed cost, marginal cost
7. revenue, cost, profit
8. break even quantity, break even point