Math 110 sec. 54 Name Answer Key
Test 4 – 100 points ver. A

For multiple choice questions, circle entire answer.
No partial credit for multiple choice.
For all others, show <u>all</u> algebraic work to receive full credit.
Some formulas which may be helpful can be found on the last page.

1. (6 pts) If $\sum_{k=1}^{43} d_k = 50$ and $\sum_{k=1}^{43} b_k = 29$, then what is $\sum_{k=1}^{43} (4d_k - 5b_k - 2)$?

$$\sum_{k=1}^{43}(4d_k - 5b_k - 2) = \sum_{k=1}^{43} 4d_k - \sum_{k=1}^{43} 5b_k - \sum_{k=1}^{43} 2$$
$$= 4\sum_{k=1}^{43} d_k - 5\sum_{k=1}^{43} b_k - \sum_{k=1}^{43} 2$$
$$= 4(50) - 5(29) - 43(2)$$
$$= 200 - 145 - 86$$
$$= \boxed{-31}$$

2. (5 pts) What is the 13th term of the following sequence:

1536, -768, 384, -192, 96, …

A. $-\frac{3}{4}$ B. $\frac{3}{8}$ C. 92 D. $-\frac{3}{8}$ E. -92

geometric w/ $r = \frac{96}{-192} = -\frac{1}{2}$

$$a_{13} = 1536\left(-\tfrac{1}{2}\right)^{12} = 1536\tfrac{1}{4096} = \tfrac{3}{8}$$

3. (7 pts) For an arithmetic sequence, if d=2 and $a_{27} = 111$, find a_1.

$$a_{27} = a_1 + (27-1)d$$
$$111 = a_1 + 26(2)$$
$$111 - 52 = a_1$$
$$\boxed{59 = a_1}$$

4. (5 pts) What is the 57th term of the following sequence:

10, 6, 2, -2, …

A. 214 B. -218 C. -112 D. 234 E. none of these

arithmetic w/ $d = 6 - 10 = -4$

$$a_{57} = a_1 + (57-1)d$$
$$= 10 + 56(-4)$$
$$= 10 - 224$$
$$= -214$$

5. (5 pts) Evaluate the expression: $\sum_{m=0}^{4}(30 - m!)$

A. 116 B. 34 C. 46 D. 66 E. 140

$$\sum_{m=0}^{4}(30 - m!) = \overbrace{30-0!}^{m=0} + \overbrace{30-1!}^{m=1} + \overbrace{30-2!}^{m=2} + \overbrace{30-3!}^{m=3} + \overbrace{30-4!}^{m=4}$$
$$= 30(5) - 1 - 1 - 2 - 6 - 24$$
$$= 150 - 34$$
$$= 116$$

6. (7 pts) Determine the value of c so that $\sum_{k=1}^{20}(k^2 - ck + 3) = 2510$

$$\sum_{k=1}^{20}(k^2 - ck + 3) = 2510$$
$$\sum_{k=1}^{20} k^2 - c\sum_{k=1}^{20} k + \sum_{k=1}^{20} 3 = 2510$$
$$\frac{20 \cdot 21 \cdot 41}{6} - c\,\frac{20 \cdot 21}{2} + 20(3) = 2510$$
$$-c(210) = 2510 - 60 - 2870$$
$$-210c = -420$$
$$c = 2$$

7. (7 pts) In a series of new experiments, Dennis the mad scientist is going to start by making a small batch of Loony Brew, then double the amount he makes on each successive experiment. He will make one gallon during his first trial. (Thus he will make 2 gallons on the second trial, 4 gallons on the third, etc.) If he makes 10 total trials, how many gallons of Loony Brew will he make overall?

$1+2+4+\ldots+2^9$

geometric with $a_1=1$, $r=2$, $n=10$

$S_{10} = 1\frac{1-2^{10}}{1-2} = \frac{1-1024}{-1} = \frac{-1023}{-1}$

$= 1023$ gallons

8. (8 pts) A certain lake is stocked with 2000 fish. The population is growing according to the logistics curve $P = \frac{20,000}{1+9e^{-t/8}}$ where t is measured in months since the lake was initially stocked.

a) What is the population of the lake after 5 months?

$P(5) = \frac{20,000}{1+9e^{-5/8}} = \frac{20,000}{1+4.817}$

$=$ **3438 fish**

b) After how many months will the fish population be 9,000? (round to the nearest month)

$9000 = \frac{20,000}{1+9e^{-t/8}}$

$9(1+9e^{-t/8}) = 20$

$1+9e^{-t/8} = 20/9$

$9e^{-t/8} = 11/9$

$e^{-t/8} = 11/81$

$\frac{-t}{8} = \ln\left(\frac{11}{81}\right)$

$t = -8\ln(11/81)$

$t = 15.97$

16 months

9. (7 pts) Express $1 + 4 + 9 + 16 + 25 + \ldots + 100 + 121 + 144$ in summation notation.

$$\sum_{k=1}^{12} k^2$$

3. (5 pts) For an arithmetic sequence, if $a_4 = 2$ and $a_9 = \frac{16}{3}$, what is d?

A. $-\frac{2}{3}$ B. $-\frac{4}{3}$ C. $-\frac{4}{9}$ D. $\frac{2}{3}$ E. None of these

$a_4 = a_1 + 3d$

$2 = a_1 + 3d$

$a_1 = 2 - 3d$

$a_9 = a_1 + 8d$

$\frac{16}{3} = a_1 + 8d$

$\frac{16}{3} = 2 - 3d + 8d$

$\frac{10}{3} = 5d$

$d = \frac{2}{3}$

11. (5 pts) Which of the following statements is/are TRUE?

I. $\sum_{k=1}^{55}(21k^2 + 28k) = 7\sum_{k=1}^{55}(3k^2 + 4k)$

II. $\sum_{j=6}^{26} 8 = 208$

III. $\sum_{k=1}^{55}(7k^2 - 3) - \sum_{k=1}^{55}(2k^2 + 4k) = \sum_{k=1}^{55}(5k^2 - 4k - 3)$

A. I only B. II & III only C. I & III only D. I & II only E. II only

12. (7 pts) Find a general formula for the n^{th} term of the following sequence:

$\frac{1}{3}, \frac{3}{9}, \frac{5}{27}, \frac{7}{81}, \ldots$ (answers are not unique)

$a_n = \frac{2n-1}{3^n}$

13. (5 pts) Simplify: $\frac{(n+2)!}{2\cdot(n-1)!}$

$\frac{(n+2)(n+1)(n)\cancel{(n-1)!}}{2\cancel{(n-1)!}}$

A. $\frac{(\frac{n}{2}+2)!}{(n-1)!}$ B. $\frac{(n+2)!}{(2n-2)!}$ C. $\frac{n(n-1)(n-2)}{2}$

D. $\frac{(n+1)!}{(n-1)!}$ E. $\frac{(n+2)(n+1)n}{2}$

14. (7 pts) Determine the following sum: $\sum_{n=1}^{11}(-2)^n$ $a_1 = -2$ $r = -2$

$$S_{11} = (-2)\frac{1-(-2)^{11}}{1-(-2)}$$
$$= (-2)\frac{1-(-2048)}{1+2}$$
$$= -2\frac{2049}{3}$$
$$= -1366$$

Name Answer Key

15. (5 pts) What is the sum of the sequence 120, 116, 112, 108, …, -36 ?

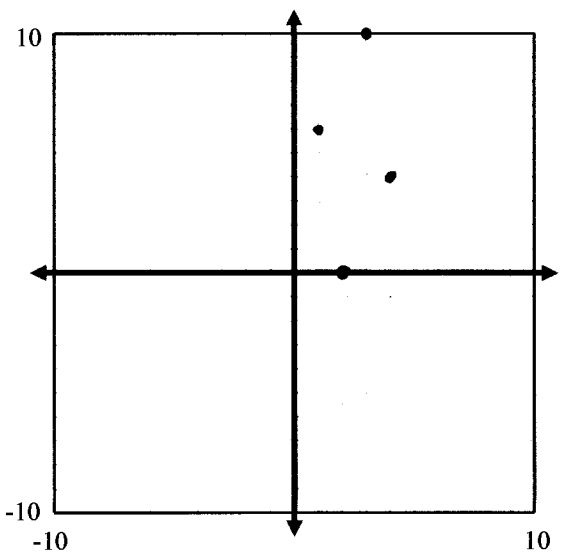
(A. 1680) B. 1638 C. 1716 D. 3120 E. 3042

arithmetic – $a_1 = 120,\ d = -4$

$$a_n = a_1 + (n-1)d$$
$$-36 = 120 + (n-1)(-4)$$
$$-156 = -4n + 4$$
$$-160 = -4n$$
$$n = 40$$

$$S_{40} = \frac{n}{2}(a_1 + a_n) = \frac{40}{2}(120 + (-36)) = 20(84)$$
$$= 1680$$

16. (4 pts) Graph the first four ordered pairs of the sequence $a_n = 2n - 4(-1)^n$



$a_1 = 2\cdot 1 - 4(-1)$
$= 2 + 4$
$= 6$

$a_2 = 2\cdot 2 - 4(1)$
$= 4 - 4$
$= 0$

$a_3 = 2\cdot 3 - 4(-1)$
$= 6 + 4$
$= 10$

$a_4 = 2\cdot 4 - 4(1)$
$= 8 - 4$
$= 4$

17. (5 pts) What are the first four terms of the sequence defined by

$$a_n = \begin{cases} 5 & \text{for } n = 1 \\ 2n + a_{n-1} & \text{for } n = 2, 3, 4, \ldots \end{cases}$$

A. 5, 9, 13, 15 B. 5, 7, 9, 11 C. 2, 6, 12, 20 (D. 5, 9, 15, 23) E. 5, 5, 5, 5

$a_1 = 5$

$a_2 = 2 \cdot 2 + 5 = 9$

$a_3 = 2 \cdot 3 + 9 = 15$

$a_4 = 2 \cdot 4 + 15 = 23$

Formulas:

$a_n = a_1 + (n-1)d$

$a_n = a_1 r^{n-1}$

$S_n = n\left(\frac{a_1 + a_n}{2}\right)$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_n = a_1\left(\frac{1-r^n}{1-r}\right),\ r \neq 1$

$\sum_{k=1}^{\infty} a_1 r^{k-1} = \frac{a_1}{1-r},\ |r| < 1$

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

FINAL EXAM:
MONDAY DECEMBER 12
ILC 120
8:00 AM