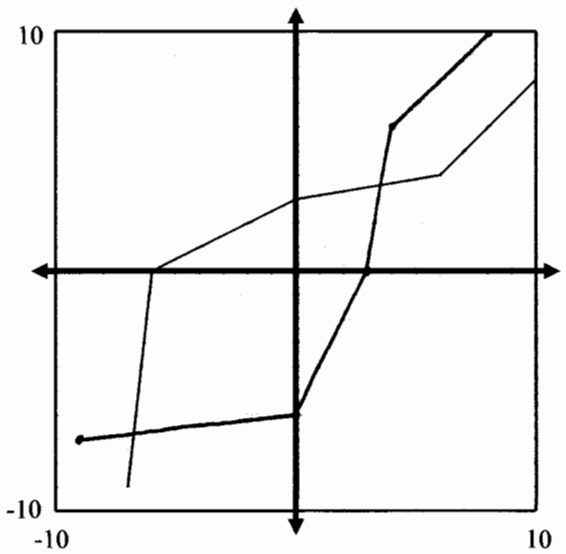
Math 110 sec. 54 Name Answer Key
Test 3 – 100 points ver. B

For multiple choice questions, circle entire answer.
No partial credit for multiple choice.
For all others, show <u>all</u> algebraic work to receive full credit.
Some formulas which may be helpful can be found on the last page.

1. (7 pts) Sketch the inverse of the function given below on the same set of axes



2. (5 pts) Which of the following is/are TRUE about the function $R(x) = \dfrac{15}{x^2 - x - 6}$?

I. $x = 15$ is a zero of $R(x)$
II. $R(x)$ has a y-intercept of $(0, \frac{5}{2})$
III. $R(x)$ has no zeroes

A. II only B. II & III only C. I & II only (D. III only) E. I only

3. (5 pts) Which of the following is/are correct? $[b > 0, b \neq 1]$

I. $\log_b 1 = 0$ II. $\log_b 0 = 0$ III. $\ln 1 = e$ IV. $\ln e^c = c$

A. I only (B. I & IV only) C. II & IV only D. III only E. IV only

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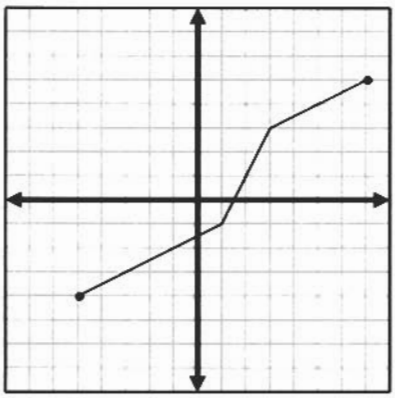
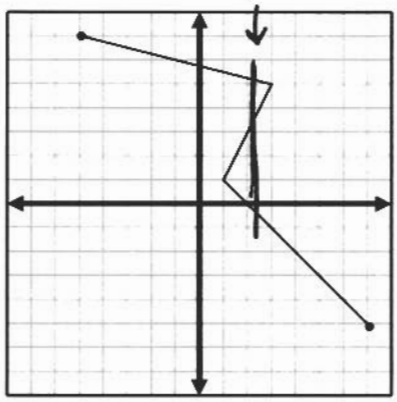
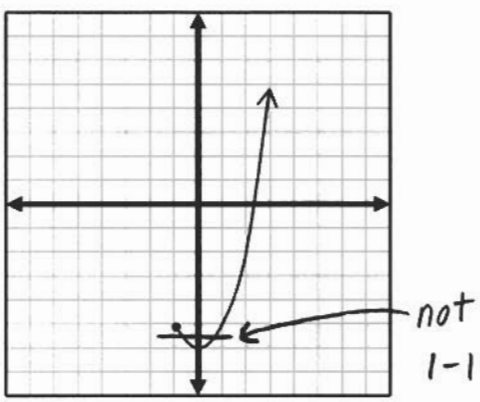
4. (5 pts) Which of the following is/are graphs of <u>one-to-one</u> <u>functions</u>?



I



II



III

A. II only B. II & III only C. I & II only D. III only (E. I only)

5. (5 pts) If $Q(x) = 19x - 18$, what is $Q(Q^{-1}(5))$?

A. $\frac{23}{9}$ B. 77 (C. 5) D. -5 E. none of these

6. (7 pts) Find and label all the asymptotes of $f(x)=\dfrac{40x^2+817x+355}{x+20}$.

$$\begin{array}{r} 40x+17 \\ x+20\overline{)\,40x^2+817x+355} \\ -40x^2-800x \\ \hline 17x+355 \\ -17x-340 \\ \hline 15 \end{array}$$

$f(x)=\dfrac{15}{x+20}+\underbrace{40x+17}_{q(x)}$

Slant asymptote: $y=40x+17$

Vertical asymptote: $x=-20$

$x+20=0$

$x=-20$

7. (7 pts) Dennis the mad scientist needs a solution of crazy juice that is at exactly 71° for his latest diabolical experiment. His only batch of crazy juice is kept stored at 125°. If he places the juice in a cooler which is kept at a constant 60°, and the rate of change is -0.018 when t is measured in minutes, how long must he wait to perform his experiment? (round to the nearest minute)

$71=60+(125-60)e^{-.018t}$

$11=65e^{-.018t}$

$\dfrac{11}{65}=e^{-.018t}$

$\ln\left(\dfrac{11}{65}\right)=-.018t$

$t=\dfrac{\ln\left(\frac{11}{65}\right)}{-.018}=98.69$

He must wait 99 minutes

8. (5 pts) Solve for x in the equation: $\log_3 x-\log_3(x-1)=5-\log_3(27)$

A. $\frac{1}{8}$ (B. $\frac{9}{8}$) C. $\frac{10}{9}$ D. $\frac{100}{99}$ E. $\frac{8}{9}$

$\log_3\left(\dfrac{x}{x-1}\right)=5-3$

$\dfrac{x}{x-1}=3^2$

$x=9(x-1)$

$x=9x-9$

$9=8x$

$x=\dfrac{9}{8}$

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9. (2 pts each) True or False. Circle the correct answer for each question.

The graphs of $f(x)=\dfrac{x^2-4}{x+2}$ and $g(x)=x-2$ are exactly identical	T	(F)
The domain of an exponential function is all real numbers	(T)	F
The functions $f(x)=x^2$ and $g(x)=\sqrt{x}$ are inverses of each other	T	(F)
The solution to $b^x=M$ is $x=\log_b M$	(T)	F
The range of a logarithmic function is all real numbers	(T)	F

10. (5 pts) If $f(x)$ is a one-to-one function and $f(4)=9$, which one of the following must be a point on the graph of $f^{-1}(x)$?

A. (-4, -9) B. $(4,\frac{1}{9})$ (C. (9, 4)) D. (4, -9) E. (-4, 9)

$f^{-1}[f(4)] = f^{-1}(9)$

$4 = f^{-1}(9)$

11. (7 pts) Approximate $\log_7 91$ to the nearest .0001.

$$\log_7 91 = \frac{\ln 91}{\ln 7} = \boxed{2.3181}$$

12. (5 pts) If $G(x)=\sqrt[3]{6-x}$, what is $G^{-1}(x)$?

A. $G^{-1}(x)=\dfrac{1}{\sqrt[3]{6-x}}$ B. $G^{-1}(x)=-\sqrt[3]{6-x}$ (C. $G^{-1}(x)=6-x^3$)

D. $G^{-1}(x)=x^3+6$ E. None of these

$G(y) = x$

$\sqrt[3]{6-y} = x$

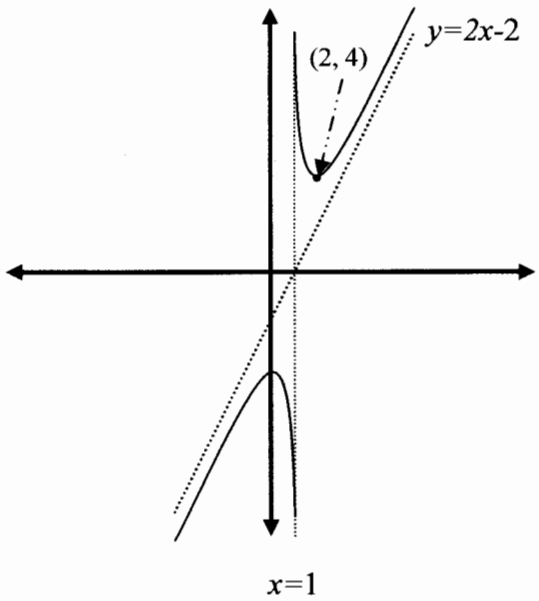
$6-y = x^3$

$f^{-1}(x) = y = 6-x^3$

13. (8 pts) Find an equation of the following rational function $f(x)$:

Hint: What type of asymptotes does it have, and what does that tell you about the equation?

Express your answer as $f(x)=\frac{n(x)}{d(x)}$.



$$f(x)=\frac{a}{x-1}+2x-2$$

$$f(2)=\frac{a}{1}+4-2=4$$

$$a+2=4$$

$$a=2$$

$$f(x)=\frac{2}{x-1}+2x-2\left(\frac{x-1}{x-1}\right)$$

$$=\frac{2+2x^2-2x-2x+2}{x-1}$$

$$\boxed{f(x)=\frac{2x^2-4x+4}{x-1}}$$

14. (7 pts) Determine <u>algebraically</u> whether or not $T(x)=\frac{5}{x^2+7}$ is a one-to-one function.

$$T(c_1)=T(c_2)$$

$$\frac{5}{c_1^2+7}=\frac{5}{c_2^2+7}$$

$$5(c_2^2+7)=5(c_1^2+7)$$

$$c_2^2+7=c_1^2+7$$

$$c_2^2=c_1^2$$

$$c_2=\pm c_1$$

So, $T(x)$ is not one-to-one.

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15. (5 pts) Express as a sum or difference of the natural logarithms of x, y, and z: $\ln\sqrt{\frac{x^3y}{z^4}}$

A. $\frac{(3\ln x)(\ln y)}{4\ln z}$

B. $\frac{3}{2}\ln x+\ln y-4\ln z$

C. $\frac{3}{2}\ln x+\frac{1}{2}\ln y-2\ln z$

D. $\ln x+2\ln y-4\ln z$

E. $\sqrt{\frac{\ln x^3+\ln y}{\ln z^4}}$

$\ln\sqrt{\frac{x^3y}{z^4}} = \ln\left(\frac{x^3y}{z^4}\right)^{1/2}$

$= \frac{1}{2}\ln\left(\frac{x^3y}{z^4}\right)$

$= \frac{1}{2}\left[\ln(x^3y) - \ln z^4\right]$

$= \frac{1}{2}\left[\ln x^3+\ln y-\ln z^4\right] = \frac{1}{2}(3\ln x+\ln y-4\ln z)$

$= \frac{3}{2}\ln x+\frac{1}{2}\ln y-2\ln z$

16. (7 pts) How much MORE money will you earn in an account that compounds interest continuously than in an account that compounds interest quarterly if you invest \$5000 for 6 years at an interest rate of 7%?

$5000e^{(.07)6} - 5000\left(1+\frac{.07}{4}\right)^{4\cdot6} = 5000\left(e^{.42}-1.0175^{24}\right)$

$= 27.594$

\$27.59

or:

continuously –

$5000\,e^{.42} = 7609.81$

quarterly –

$5000\,(1.0175)^{24} = 7582.21$

$7609.81 - 7582.21 = \$27.60$

Formulas:

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$A(t) = C+(A_0-C)e^{rt}$$