

**For multiple choice questions, circle entire answer. No partial credit for multiple choice.  
For all others, show all algebraic work to receive full credit.  
NO CALCULATORS ALLOWED ON THIS EXAM!!**

1. (2 pts each) Match the transformation of the function  $y = f(x)$  to its description in words:

- |                                   |  |
|-----------------------------------|--|
| <u>C</u> 1. $y = f(x) - 5$        | The graph of $f(x)$ is                                   |
| <u>G</u> 2. $y = 4f(x)$           | A) reflected across the $y$ -axis                        |
| <u>A</u> 3. $y = f(-x)$           | B) reflected across the $x$ -axis                        |
| <u>D</u> 4. $y = f(x+5)$          | C) shifted down 5 units                                  |
| <u>H</u> 5. $y = f(\frac{1}{4}x)$ | D) shifted left 5 units                                  |
|                                   | E) shifted up 5 units                                    |
|                                   | F) shifted right 5 units                                 |
|                                   | G) a vertical expansion by a factor of 4                 |
|                                   | H) a horizontal expansion by a factor of 4               |
|                                   | I) a vertical compression by a factor of $\frac{1}{4}$   |
|                                   | J) a horizontal compression by a factor of $\frac{1}{4}$ |

2. (5 pts) Which one of the following is a polynomial of lowest degree having zeros 7, 0, 1 and -2, where 1 has multiplicity 2, and -2 has multiplicity 3?

- A.  $f(x) = x(x-1)^2(x-7)(x+2)^3$       B.  $f(x) = x(x-1)^2(x+7)(x-2)^3$   
C.  $f(x) = x(x-7)(x+2)^3$       D.  $f(x) = x(x+7)(x+1)^2(x-2)^3$       E.  $f(x) = (x-1)^2(x-7)(x+2)^3$

3. (5 pts) What is the degree, leading coefficient and constant term of the polynomial

$$P(x) = x(2x-2)^2(5x+3)(x+1)^3 ?$$

leading term:  $x(4x^2)(5x)(x^3) = 20x^3x^4 = 20x^7$

- A. degree is 7, leading coefficient is 10, constant term is -6  
B. degree is 7, leading coefficient is 20, constant term is 0  
C. degree is 3, leading coefficient is 10, constant term is 6  
D. degree is 7, leading coefficient is 20, constant term is 12  
E. degree is 4, leading coefficient is 10, constant term is 0  
F. degree is 3, leading coefficient is 20, constant term is -12

constant term:  $0(-2)^2(3)(1)^3 = 0$

4. (5 pts each) Use  $f(x) = 3x - x^2$  and  $g(x) = x + 2$  to find and simplify:

a)  $(f \circ g)(x) = f(g(x))$   
 $= f(x+2)$   
 $= 3(x+2) - (x+2)^2$   
 $= 3x + 6 - (x^2 + 4x + 4)$   
 $= 3x + 6 - x^2 - 4x - 4 = -x^2 - x + 2$

$(f \circ g)(x) = -x^2 - x + 2$

b)  $(f+g)(4)$

$(f+g)(4) = f(4) + g(4)$   
 $= 3 \cdot 4 - 4^2 + 4 + 2$   
 $= 12 - 16 + 4 + 2$

$(f+g)(4) = 2$

5. (5 pts) Which of the following is/are equivalent to:  
“ $x+a$  is a factor of the polynomial  $f(x)$ ”?

- I.  $x = -a$  is a zero of  $f(x)$
- II.  $(-a, 0)$  is an  $x$ -intercept of  $f(x)$
- III.  $x = a$  is a solution of  $f(x) = 0$

A. II only

B. II & III only

C. I & II only

D. III only

E. I only

6. (2 pts each) True or False. Circle the correct answer for each question.

The domains of  $(f+g)(x)$ ,  $(f-g)(x)$ ,  $(fg)(x)$ , and  $\left(\frac{f}{g}\right)(x)$   
are always equal

T **F**

The graph of a degree 7 polynomial has at least 6 turning points

T **F**

The graph of  $f(x) = ax^2 + bx + c$  with  $a > 0$  has a maximum

T **F**

The graph of a degree 4 polynomial has at most 4  $x$ -intercepts

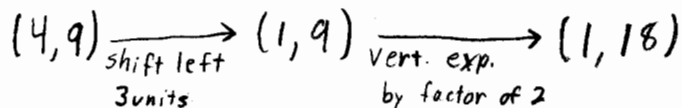
**T** F

The lines  $2y = x + 9$  and  $y = -\frac{1}{2}x + 3$  are perpendicular

T **F**

7. (5 pts) If (4, 9) is a point on the graph of  $f(x)$ , which one of the following must be a point on the graph of  $y = 2f(x+3)$ ?

- A. (2, 18)      **B. (1, 18)**      C. (14, 9)      D. (2, 9)      E. (7, 18)



8. (6 pts) Find functions  $f(x)$  and  $g(x)$  such that  $(g \circ f)(x) = \frac{x^4}{x^4 - 2}$ . Do not use  $f(x) = x$  or  $g(x) = x$ .

(There are multiple correct answers)

$$g(x) = \frac{x}{x-2} \quad \text{or} \quad g(x) = \frac{x^2}{x^2-2} \quad \text{or} \quad g(x) = \frac{x+2}{x} \quad \text{or} \dots$$

$$f(x) = x^4 \quad f(x) = x^2 \quad f(x) = x^4 - 2$$

9. (6 pts) Determine the equation of the line passing through the points (-1, 6) and (1, 3). Give your answer in slope-intercept form.

$$m = \frac{6-3}{-1-1} = \frac{3}{-2}$$

$$y = -\frac{3}{2}x + b$$

$$3 = -\frac{3}{2}(1) + b$$

$$3 + \frac{3}{2} = b \rightarrow b = \frac{9}{2}$$

$$y = -\frac{3}{2}x + \frac{9}{2}$$

10. (6 pts) Rewrite the quadratic  $f(x) = -2x^2 - 10x + \frac{9}{2}$  in standard form. (complete the square)

$$f(x) = -2x^2 - 10x + \frac{9}{2}$$

$$= -2(x^2 + 5x) + \frac{9}{2}$$

$$= -2(x^2 + 5x + (\frac{5}{2})^2) + \frac{9}{2} - (-2)(\frac{5}{2})^2$$

$$= -2(x + \frac{5}{2})^2 + \frac{9}{2} + 2(\frac{25}{4})$$

$$f(x) = -2(x + \frac{5}{2})^2 + 17$$

11. (5 pts) Determine the point-slope form of the equation of the line through (4, 1) and parallel to  $2y - x = -6$ .

$$2y = x - 6$$

$$y = \frac{1}{2}x - 3$$

$$m = \frac{1}{2}$$

$$y = \frac{1}{2}(x - 4) + 1$$

12. (5pts) Find the vertex of  $f(x) = x^2 - 6x + 7$ .

$$\text{Vertex: } \left( \frac{-b}{2a}, c - \frac{b^2}{4a} \right) = \left( \frac{-(-6)}{2}, 7 - \frac{(-6)^2}{4} \right)$$

$$= \left( \frac{6}{2}, 7 - \frac{36}{4} \right) = (3, -2)$$

The vertex is located:

A. Above the x-axis

B. Below the x-axis

C. At the origin

D. On the x-axis but not at the origin

E. None of these

13. (6pts) Find the equation of the parabola which passes through the point (12, 14) and whose vertex is (9, -4). Write your answer in the general form.

$$y = a(x - 9)^2 - 4$$

$$14 = a(12 - 9)^2 - 4$$

$$18 = a(3)^2$$

$$\frac{18}{9} = a$$

$$a = 2$$

$$y = 2(x - 9)^2 - 4$$

$$= 2(x^2 - 18x + 81) - 4$$

$$y = 2x^2 - 36x + 158$$

14. (5pts) Which one of the following is a factor of  $Q(x) = x^3 + 7x^2 + 19x + 18$ ?

A.  $x - 2$

B.  $x + 3$

C.  $x - 3$

D.  $x + 2$

E. none of these

Can't be  $x - 2$  or  $x - 3$  since all the coefficients are positive.

$$\begin{array}{r|rrrr} -2 & 1 & 7 & 19 & 18 \\ & & -2 & -10 & -18 \\ \hline & 1 & 5 & 9 & 0 \end{array}$$

$$\Rightarrow x^3 + 7x^2 + 19x + 18 = (x + 2)(x^2 + 5x + 9)$$

15. (6pts) Dennis the mad scientist needs exactly 550 ml of a 65% solution of crazy juice for his latest diabolical experiment. He has only a 30% solution and an 85% solution to mix. How much of the 30% concentrate crazy juice should he use?

$$.3x + .85(550 - x) = .65(550)$$

$$.3x + 467.5 - .85x = 357.5$$

$$110 - .55x = 0$$

$$110 = .55x$$

$$x = \frac{110}{.55}$$

$$x = 200$$

He needs 200 ml of the 30% concentrate.

0.05 x (550) =	27.5
0.10 x (550) =	55
0.15 x (550) =	82.5
0.20 x (550) =	110
0.25 x (550) =	137.5
0.30 x (550) =	165
0.35 x (550) =	192.5
0.40 x (550) =	220
0.45 x (550) =	247.5
0.50 x (550) =	275
0.55 x (550) =	302.5
0.60 x (550) =	330
0.65 x (550) =	357.5
0.70 x (550) =	385
0.75 x (550) =	412.5
0.80 x (550) =	440
0.85 x (550) =	467.5
0.90 x (550) =	495
0.95 x (550) =	522.5

16. (6pts) What is the remainder when  $x^4 + 2x^3 + 2x^2 + x + 9$  is divided by  $x^2 - x + 1$ ?

$$\begin{array}{r}
 x^2 + 3x + 4 \\
 x^2 - x + 1 \overline{) x^4 + 2x^3 + 2x^2 + x + 9} \\
 \underline{-x^4 + x^3 - x^2} \phantom{+ x + 9} \\
 3x^3 + x^2 + x \phantom{+ 9} \\
 \underline{-3x^3 + 3x^2 - 3x} \phantom{+ 9} \\
 4x^2 - 2x + 9 \\
 \underline{-4x^2 + 4x - 4} \\
 \hline
 2x + 5
 \end{array}$$

The remainder is  $2x + 5$