

Section 10.2

1. Find the first four nonzero terms of the Taylor series for $\frac{1}{\sqrt{1+x}}$ about $x = 0$.
2. Find the first four nonzero terms of the Taylor series for $\tan^{-1} x$ about $x = 1$.
3. Find the exact sum of the following series:

$$\frac{\pi}{6} - \frac{\pi^3}{6^3 \cdot 3!} + \frac{\pi^5}{6^5 \cdot 5!} - \frac{\pi^7}{6^7 \cdot 7!} + \frac{\pi^9}{6^9 \cdot 9!} - \cdots$$

4. What is the sum of the alternating harmonic series, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$? (You should recognize this as a Taylor series evaluated at a particular value of x).

Section 10.3

Find the Taylor series expansions of the given functions about $x = 0$. Give (at least) the first 3 nonzero terms and the general term. For example, your answers should look like:

$$\cos(x^2) = 1 - \frac{x^4}{2!} + \frac{x^8}{4!} + \cdots + (-1)^n \frac{x^{4n}}{(2n)!} + \cdots$$

1. $\frac{z^3}{e^{z^2}}$
2. $\ln(1 - 3t)$
3. What is the Taylor series expansion of $2 \sin x \cos x$ about $x = 0$? [Hint: it may help to use a trig identity]
4. Find the Taylor series expansion about $x = 0$ for $\sin^2 x$ from the series for $2 \sin x \cos x$.
5. Find the Taylor series expansion about $x = 0$ for $\cos^2 x$.