This is meant to serve as an aid in studying for test 2. It is not meant to be an exhaustive list of material or of possible test questions.

Determine if the following integrals converge or diverge. Be sure to explain *completely* why you can make your conclusion of convergence or divergence. Also, if it converges, find its value, or give an upper bound.

1. $\displaystyle\int_1^\infty \frac{\sin(3x)+3}{x^3+x}dx$
2. $\displaystyle\int_1^\infty e^{-x^2}\,dx$
3. $\displaystyle\int_1^\infty \frac{d\theta}{\sqrt{\theta^3+\theta}}$
4. $\displaystyle\int_{-\infty}^0 \frac{1}{(x-8)^{2/3}}dx$
5. $\displaystyle\int_{-\infty}^{-1} \frac{1}{-x^4-x^2}dx$
6. $\displaystyle\int_3^\infty \frac{1}{x^2-1}dx$
7. $\displaystyle\int_0^3 \frac{x}{x^2-1}dx$
8. $\displaystyle\int_{-\infty}^{-2} \frac{1}{x^3}dx$
9. $\displaystyle\int_0^\infty \frac{x}{1+x^2}dx$
10. Find the arc length of $y=\frac{2}{3}x^{2/3}$ from $(1,\frac{2}{3})$ to $(8,\frac{8}{3})$.
11. Find the arc length of $y=\dfrac{x^3}{12}+\dfrac{1}{x}$ from $(1,\frac{13}{12})$ to $(2,\frac{7}{6})$.
12. Find the arc length of $y=\ln x$ from $(1,0)$ to $(4,\ln 4)$.
13. Find the length of the curve defined by $x=5t^2$ and $y=2t^3$ from $t=0$ to $t=1$.
14. Find the length of the curve defined by $x=e^t\cos t$ and $y=e^t\sin t$ for $0\le t\le \pi/2$.
15. Find the length of the curve defined by $x=t\cos t-\sin t$ and $y=t\sin t+\cos t$ from $t=0$ to $t=\pi/2$.
16. Derive the formula for the circumference of a circle by finding the length of the arc given by $y=\sqrt{r^2-x^2}$ from $x=-r$ to $x=r$.
17. A small water tank is shaped as follows: It has a cylidrical base, of radius 1 foot and height 2 feet, with a cone on top of the cylinder, of height 2 feet, where the base of the cone sits on top of the cylinder. So, the total height is 4 feet. The water has dirt in it, though, making it more dense at the bottom of the tank than at the top. Its density y feet from the top of the tank is $y+63$ lbs/ft^3. What is the total weight of the dirt in the tank?
18. In the setup above, find the total work required to pump the water out of the top of the tank.

19. A water tank is to be constructed in the shape of the "middle part" of a sphere. That is, the tank will be a sphere of radius 8 feet, but 2 feet will be cut off the bottom and the top of the sphere forming a flat base and a flat top. What is the maximum amount of water the the tank can hold (in ft^3)?

20. If the water tank in the previous question was to hold a liquid whose density h feet above the bottom of the tank is given by $\delta(h) = 80 - 2h$ lb/ft^3, what is the total weight of the liquid in the tank?

21. Given the setup of the previous problem, how much work is required to pump out all of the liquid in a full tank?

22. Given the same setup, how much is required to pump out all of the liquid if the tank began only half full?

23. Again with the same setup, how much work is required to pump out a full tank, if it must be pumped to a height of 2 feet above the top of the tank?

24. Use slices and a definite integral to compute the volume of a right circular cone of base radius r and height h.

25. Consider the region in the xy-plane bounded by $y = x^2 - 7x - 1$ $y = -x^2 + 9x + 1$, the y-axis, and $x = 8$.
    What is the volume of a solid whose base is this region and whose cross-sections perpendicular to the x-axis are squares?

26. What if the cross-sections perpendicular to the x-axis are semi-circles?

27. What if the cross-sections perpendicular to the x-axis are rectangles whose height is twice their base?

28. What if the cross-sections perpendicular to the x-axis are rectangles with a fixed area of 400?

29. What if the region is rotated around the y-axis? [This one is difficult]

1 Converges; upper bound is 2 [by comparing it to $\frac{4}{x^3}$]

2 Converges; upper bound is $1/e$ [by comparing it to e^{-x}]

3 Converges; upper bound is 2 [by comparing it to $\frac{1}{\theta^{3/2}}$]

4 diverges

5 converges; $\pi/4 - 1$

6 converges; $\frac{1}{2}\ln 2$

7 diverges (computation; no need for a comparison)

8 converges; -1/8

9 diverges (computation; no need for a comparison)

10 $\left(4 + \frac{16}{81}\right)^{3/2} - \left(1 + \frac{16}{81}\right)^{3/2} \approx 7.29$

11 13/12

12 ≈ 3.34

13 $\frac{2}{27}\left(34^{3/2} - 125\right) \approx 5.43$

14 $\sqrt{2}\left(e^{\pi/2} - 1\right) \approx 5.39$

15 $\pi^2/8 \approx 1.23$

16 $C = 2\pi r$

17 $43\pi/5 \approx 27$ lb [The total weight of the muddied water is $175\pi \approx 549.8$ lb

18 ≈ 756.3 ft-lb

19 624π

20 $42432\pi \approx 133304$ lb

21 ≈ 838187 ft-lb

22 ≈ 627193 ft-lb

23 ≈ 1104795 ft-lb

24 $V = \frac{1}{3}\pi r^2 h$

25 76256/15

26 $9532\pi/15$

27 152512/15

28 3200

29