

For 1-5, (a) Show that the function f determined by the n th term of the series satisfies the hypotheses of the integral test. (b) Use the integral test to determine whether the series converges or diverges.

$$1. \sum_{n=1}^{\infty} \frac{1}{(3+2n)^2}$$

$$4. \sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$$

$$2. \sum_{n=1}^{\infty} \frac{1}{4n+7}$$

$$5. \sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$$

$$3. \sum_{n=3}^{\infty} \frac{\ln n}{n}$$

Use a basic comparison to determine whether the series converges or diverges.

$$6. \sum_{n=1}^{\infty} \frac{1}{n^4+n^2+1}$$

$$8. \sum_{n=1}^{\infty} \frac{\arctan n}{n}$$

$$7. \sum_{n=1}^{\infty} \frac{1}{n3^n}$$

$$9. \sum_{n=1}^{\infty} \frac{1}{n^n}$$

Use the limit comparison test to determine whether the series converges or diverges.

$$10. \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$$

$$12. \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+9}$$

$$11. \sum_{n=2}^{\infty} \frac{1}{\sqrt{4n^3-5n}}$$

Find $\lim_{n \rightarrow \infty} (a_{n+1}/a_n)$, and use the ratio test to determine if the series converges or diverges or if the test is inconclusive.

$$13. \sum_{n=1}^{\infty} \frac{3n+1}{2^n}$$

$$16. \sum_{n=1}^{\infty} \frac{n+3}{n^2+2n+5}$$

$$14. \sum_{n=1}^{\infty} \frac{5^n}{n3^{n+1}}$$

$$17. \sum_{n=1}^{\infty} \frac{n!}{e^n}$$

$$15. \sum_{n=1}^{\infty} \frac{100^n}{n!}$$

Determine whether the series converges or diverges.

$$18. \sum_{n=1}^{\infty} \frac{2n + n^2}{n^3 + 1}$$

$$25. \sum_{n=1}^{\infty} \frac{(2n + 1)^3}{(n^3 + 1)^2}$$

$$19. \sum_{n=1}^{\infty} \frac{n^{10} + 10}{n!}$$

$$26. \sum_{n=1}^{\infty} \frac{n^2 + 2^n}{n + 3^n}$$

$$20. \sum_{n=1}^{\infty} \frac{1 + 2^n}{1 + 3^n}$$

$$27. \sum_{n=1}^{\infty} \frac{n!}{(n + 1)^5}$$

$$21. \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{5n^2 + 1}}$$

$$28. \sum_{n=1}^{\infty} ne^{-n^2}$$

$$22. \sum_{n=1}^{\infty} \frac{3^n}{n^2 + 4}$$

$$29. \sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3 + 1}}$$

$$23. \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{2n + 1}}$$

$$30. \sum_{n=1}^{\infty} \frac{\ln n}{n^3}$$

$$24. \sum_{n=1}^{\infty} ne^{-n}$$

$$31. \sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n(n + 1)}$$

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$32. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{2n + 1}}$$

$$38. \sum_{n=1}^{\infty} \frac{\cos \frac{1}{6}\pi n}{n^2}$$

$$33. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n + 1)}$$

$$39. \sum_{n=1}^{\infty} (-1)^n n \sin \frac{1}{n}$$

$$34. \sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$$

$$40. \sum_{n=2}^{\infty} (-1)^n \frac{1}{n\sqrt{\ln n}}$$

$$35. \sum_{n=1}^{\infty} (-1)^n \frac{5}{n^3 + 1}$$

$$41. \sum_{n=1}^{\infty} \frac{n^n}{(-5)^n}$$

$$36. \sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$$

$$42. \sum_{n=1}^{\infty} (-1)^n \frac{1 + 4^n}{1 + 3^n}$$

$$37. \sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 3}{(2n - 5)^2}$$

$$43. \sum_{n=1}^{\infty} (-1)^n \frac{\cos \pi n}{n}$$

44.
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n-4)^2 + 5}$$

45.
$$\sum_{n=1}^{\infty} (-1)^n \frac{2n^4}{e^n}$$

Find the interval of convergence of the power series.

46.
$$\sum_{n=0}^{\infty} \frac{1}{n+4} x^n$$

52.
$$\sum_{n=0}^{\infty} \frac{1}{(-4)^n} x^{2n+1}$$

47.
$$\sum_{n=0}^{\infty} \frac{n^2}{2^n} x^n$$

53.
$$\sum_{n=0}^{\infty} \frac{2^n}{(2n)!} x^{2n}$$

48.
$$\sum_{n=0}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} x^n$$

54.
$$\sum_{n=0}^{\infty} \frac{3^{2n}}{n+1} (x-2)^n$$

49.
$$\sum_{n=2}^{\infty} \frac{n}{n^2+1} x^n$$

55.
$$\sum_{n=0}^{\infty} \frac{n^2}{2^{3n}} (x+4)^n$$

50.
$$\sum_{n=2}^{\infty} \frac{\ln n}{n^3} x^n$$

56.
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{n+1} (x-3)^n$$

51.
$$\sum_{n=0}^{\infty} \frac{n+1}{10^n} (x-4)^n$$

57.
$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n6^n} (2x-1)^n$$

Hints

- 6 $b_n = \frac{1}{n^4}$
- 7 $b_n = \frac{1}{3^n}$
- 8 $b_n = \frac{\pi/4}{n}$
- 9 $b_n = \frac{1}{n^2}$
- 10 $b_n = \frac{1}{\sqrt{n}}$
- 11 $b_n = \frac{1}{n^{3/2}}$
- 12 $b_n = \frac{1}{\sqrt{n}}$
- 18 Limit comparison
- 19 Ratio test
- 20 Basic/geometric
- 21 Limit comparison
- 22 Ratio test or simply n^{th} -term
- 23 Limit comparison
- 24 Ratio
- 25 Limit
- 26 Limit
- 27 Ratio
- 28 Integral
- 29 Limit comparison with $b_n = 1/n^{1/2}$
- 30 Basic comparison ($b_n = \frac{1}{n^2}$)
- 31 Ratio
- 32 Alternating Series Test/Basic comparison
- 33 AST/Basic
- 34 n^{th} -term
- 35 AST/Basic
- 36 Ratio
- 37 n^{th} -term
- 38 Basic comparison
- 39 n^{th} -term
- 40 AST/Integral
- 41 n^{th} -term
- 42 n^{th} -term
- 43 It's the harmonic series!
- 44 AST/whatever
- 45 Ratio

Answers

- 1** (a) $f'(x) = \frac{-4}{(2x+3)^3} < 0$ if $x \geq 1$ **29** D
 (b) $\int_1^\infty f(x)dx = \frac{1}{10}$; C **30** C
2 (a) $f'(x) = \frac{-4}{(4x+7)^2} < 0$ if $x \geq 1$ **31** C
 (b) $\int_1^\infty f(x)dx = \infty$; D **32** CC
3 (a) $f'(x) = \frac{1-\ln x}{x^2} < 0$ if $x \geq 3$ **33** CC
 (b) $\int_3^\infty f(x)dx = \infty$; D **34** D
4 (a) $f'(x) = \frac{1-2x^2}{x^2(x^2-1)^{3/2}} < 0$ if $x \geq 2$ **35** AC
 (b) $\int_2^\infty f(x)dx = \frac{\pi}{6}$; C **36** AC
5 (a) $f'(x) = \frac{1-2x \arctan x}{(1+x^2)^2} < 0$ if $x \geq 1$ **37** D
 (b) $\int_1^\infty f(x)dx = \frac{3\pi^2}{32}$; C **38** AC
6 C; A possible choice for b_n is $b_n = \frac{1}{n^4}$ **39** D; recall $\sin x \approx x$ near $x = 0$, so
7 C; A possible choice for b_n is $b_n = \frac{1}{3^n}$ $\sin(\frac{1}{n})$ behaves like $\frac{1}{n}$ as $n \rightarrow \infty$, and thus
8 D; A possible choice for b_n is $b_n = \frac{\pi/4}{n}$ $\lim_{n \rightarrow \infty} n \sin(\frac{1}{n}) = 1$
9 C; A possible choice for b_n is $b_n = \frac{1}{n^2}$ **40** CC
10 D; A possible choice for b_n is $b_n = \frac{1}{\sqrt{n}}$ **41** D
11 C; A possible choice for b_n is $b_n = \frac{1}{n^{3/2}}$ **42** D
12 D; A possible choice for b_n is $b_n = \frac{1}{\sqrt{n}}$ **43** D
13 $\frac{1}{2}$; C **44** AC
14 $\frac{5}{3}$; D **45** AC
15 0; C **46** $[-1, 1)$
16 1; inconclusive **47** $(-2, 2)$
17 ∞ ; D **48** $(-1, 1]$
18 D **49** $[-1, 1)$
19 C **50** $[-1, 1]$
20 C **51** $(-6, 14)$
21 D **52** $(-2, 2)$
22 D **53** $(-\infty, \infty)$
23 D **54** $[\frac{17}{9}, \frac{19}{9})$
24 C **55** $(-12, 4)$
25 C **56** converges only for $x = 3$
26 C **57** $(-\frac{5}{2}, \frac{7}{2}]$
27 D
28 C