Determine if the following integrals converge or diverge. Be sure to explain completely why you can make your conclusion of convergence or divergence. Also, if it converges, find its value, or give an upper bound.

1. $\displaystyle\int_1^\infty \frac{\sin(3x)+3}{x^3+x}dx$
2. $\displaystyle\int_1^\infty e^{-x^2}\,dx$
3. $\displaystyle\int_1^\infty \frac{d\theta}{\sqrt{\theta^3+\theta}}$
4. $\displaystyle\int_{-\infty}^0 \frac{1}{(x-8)^{2/3}}dx$
5. $\displaystyle\int_{-\infty}^{-1} \frac{1}{-x^4-x^2}dx$
6. $\displaystyle\int_3^\infty \frac{1}{x^2-1}dx$
7. $\displaystyle\int_0^3 \frac{x}{x^2-1}dx$
8. $\displaystyle\int_{-\infty}^{-2} \frac{1}{x^3}dx$
9. $\displaystyle\int_0^\infty \frac{x}{1+x^2}dx$

Answers

1 Converges; upper bound is 2 [by comparing it to $\frac{4}{x^3}$]

2 Converges; upper bound is $1/e$ [by comparing it to e^{-x}]

3 Converges; upper bound is 2 [by comparing it to $\frac{1}{\theta^{3/2}}$]

4 diverges

5 converges; $\pi/4 - 1$

6 converges; $\frac{1}{2}\ln 2$

7 diverges (computation; no need for a comparison)

8 converges; -1/8

9 diverges (computation; no need for a comparison)