Name ______________________

Homework 21

Section 18.4

1. (4) Let C denote the path around the square with vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$, traversed counterclockwise. Compute $\oint_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = e^y\vec{i} + 2xe^y\vec{j}$.

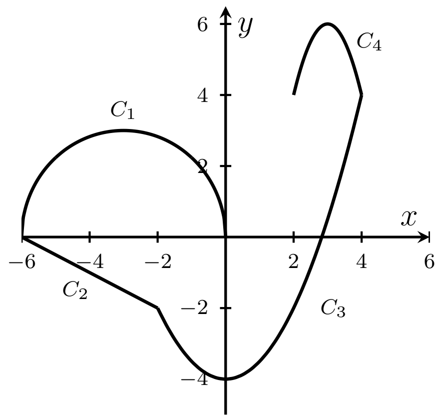
2. (4) Show that the line integral of $\vec{F} = x\vec{j}$ around a closed curve C, in the xy-plane, oriented as in Green's Theorem, gives the area of the region enclosed by the curve. Show that the line integral of $\vec{F} = -y\vec{i}$ gives the same result.

3. (6) Use the result of the previous problem to calculate the area bounded by the piecewise smooth curve consisting of the folium of Descartes

$$x = \frac{3t}{t^3+1}, \quad y = \frac{3t^2}{t^3+1}; \quad 0 \le t \le 1$$

and the line segment from $(\frac{3}{2}, \frac{3}{2})$ to $(0, 0)$.

4. (6) Consider the vector field $\vec{F} = \left(\dfrac{y - x^2y - 2xy^2}{(1+x^2)^2} \right) \vec{i} + \left(\dfrac{2y + x}{1 + x^2} \right) \vec{j}$. Does the curl test apply? Determine $\int_C \vec{F} \cdot d\vec{r}$, where C is the path $C_1 + C_2 + C_3 + C_4$ from $(0, 0)$ to $(2, 4)$ shown below.



C_1 is a semicircle.

C_2 is a line segment.

C_3 and C_4 are portions of graphs of quadratic functions.