For 1-5, (a) Show that the function f determined by the nth term of the series satisfies the hypotheses of the integral test. (b) Use the integral test to determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(3+2n)^2}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n+7}$
3. $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n}$
4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{1+n^2}$

Use a basic comparison to determine whether the series converges or diverges.

6. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4+n^2+1}$
7. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n3^n}$
8. $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan n}{n}$
9. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

Use the limit comparison test to determine whether the series converges or diverges.

10. $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+4}$
11. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{4n^3-5n}}$
12. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+9}$

Find $\lim_{n\to\infty}(a_{n+1}/a_n)$, and use the ratio test to determine if the series converges or diverges or if the test is inconclusive.

13. $\displaystyle\sum_{n=1}^{\infty} \frac{3n+1}{2^n}$
14. $\displaystyle\sum_{n=1}^{\infty} \frac{5^n}{n3^{n+1}}$
15. $\displaystyle\sum_{n=1}^{\infty} \frac{100^n}{n!}$
16. $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{n^2+2n+5}$
17. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^n}$

Determine whether the series converges or diverges.

18. $\displaystyle\sum_{n=1}^{\infty} \frac{2n+n^2}{n^3+1}$

19. $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}+10}{n!}$

20. $\displaystyle\sum_{n=1}^{\infty} \frac{1+2^n}{1+3^n}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{5n^2+1}}$

22. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^2+4}$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{2n+1}}$

24. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

25. $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)^3}{(n^3+1)^2}$

26. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+2^n}{n+3^n}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{(n+1)^5}$

28. $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{3n}{\sqrt{n^3+1}}$

30. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{2^{n-1}}{5^n(n+1)}$

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

32. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{2n+1}}$

33. $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\ln(n+1)}$

34. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln n}$

35. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5}{n^3+1}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{(-10)^n}{n!}$

37. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3}{(2n-5)^2}$

38. $\displaystyle\sum_{n=2}^{\infty} (-1)^n \frac{1}{n\sqrt{\ln n}}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(-5)^n}$

40. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1+4^n}{1+3^n}$

41. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\cos(\pi n)}{n}$

42. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{(n-4)^2+5}$

43. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2n^4}{e^n}$

Hints

6 $b_n = \frac{1}{n^4}$

7 $b_n = \frac{1}{3^n}$

8 $b_n = \frac{\pi/4}{n}$

9 $b_n = \frac{1}{n^2}$

10 $b_n = \frac{1}{\sqrt{n}}$

11 $b_n = \frac{1}{n^{3/2}}$

12 $b_n = \frac{1}{\sqrt{n}}$

18 Limit comparison

19 Ratio test

20 Basic/geometric

21 Limit comparison

22 Ratio test or simply n^{th}-term

23 Limit comparison

24 Ratio

25 Limit

26 Limit

27 Ratio

28 Integral

29 Limit comparison with $b_n = 1/n^{1/2}$

30 Basic comparison ($b_n = \frac{1}{n^2}$)

31 Ratio

32 Alternating Series Test/Basic comparison

33 AST/Basic

34 nth-term

35 AST/Basic

36 Ratio

37 nth-term

38 AST/Integral

39 nth-term

40 nth-term

41 It's the harmonic series!

42 AST/whatever

43 Ratio

Answers

1 (a) $f'(x) = \frac{-4}{(2x+3)^3} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)dx = \frac{1}{10}$; C

2 (a) $f'(x) = \frac{-4}{(4x+7)^2} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)dx = \infty$; D

3 (a) $f'(x) = \frac{1-\ln x}{x^2} < 0$ if $x \geq 3$
(b) $\int_3^\infty f(x)dx = \infty$; D

4 (a) $f'(x) = \frac{1-2x^2}{x^2(x^2-1)^{3/2}} < 0$ if $x \geq 2$
(b) $\int_2^\infty f(x)dx = \frac{\pi}{6}$; C

5 (a) $f'(x) = \frac{1-2x\arctan x}{(1+x^2)^2} < 0$ if $x \geq 1$
(b) $\int_1^\infty f(x)dx = \frac{3\pi^2}{32}$; C

6 C; A possible choice for b_n is $b_n = \frac{1}{n^4}$

7 C; A possible choice for b_n is $b_n = \frac{1}{3^n}$

8 D; A possible choice for b_n is $b_n = \frac{\pi/4}{n}$

9 C; A possible choice for b_n is $b_n = \frac{1}{n^2}$

10 D; A possible choice for b_n is $b_n = \frac{1}{\sqrt{n}}$

11 C; A possible choice for b_n is $b_n = \frac{1}{n^{3/2}}$

12 D; A possible choice for b_n is $b_n = \frac{1}{\sqrt{n}}$

13 $\frac{1}{2}$; C

14 $\frac{5}{3}$; D

15 0; C

16 1; inconclusive

17 ∞; D

18 D

19 C

20 C

21 D

22 D

23 D

24 C

25 C

26 C

27 D

28 C

29 D

30 C

31 C

32 CC

33 CC

34 D

35 AC

36 AC

37 D

38 CC

39 D

40 D

41 D

42 AC

43 AC