1. (10 $\frac{\text{pts}}{}$) Consider the function $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = \lceil \frac{x}{3} \rceil - 2$. Determine whether f is one-to-one, onto, both, or neither.

2. (8 $\frac{\text{pts}}{}$) Prove that if m and n are integers and $3m - 5n$ is odd, then $m + n$ is odd.

3. (10 $\frac{\text{pts}}{}$) Show that the following are loop invariants for the loop

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While 1 ≤ m ≤ 14178
    m := 2m
    n := 3n
End-while
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(a) $n^2 \geq m^3$

(b) $2m^6 < n^4$

4. (8 $\frac{\text{pts}}{}$) Prove that $11^n - 4^n$ is divisible by 7 for all $n \in \mathbb{Z}^+$.

5. (18 $\frac{\text{pts}}{}$) Consider the sets $A = \{a, b\}$, $B = \{b, c, d, e\}$, $X = A \cup B$, $Y = A \cap B$, and $Z = \{A, B\}$. Determine the cardinality of the following sets.

(a) X

(b) Y

(c) $A \oplus B$

(d) $A \cup Z$

(e) $\mathcal{P}(A)$

(f) $\mathcal{P}(B)$

(g) $\mathcal{P}(X)$

(h) $\mathcal{P}(Y)$

(i) $\mathcal{P}(Z)$

6. (20 $\frac{\text{pts}}{}$) Determine whether the following relations on $\mathbb{Z}$ are reflexive, anti-reflexive, symmetric, anti-symmetric, and/or transitive.

(a) $(m, n) \in R$ if $\max\{m, n\} = 5$

(b) $m \sim n$ if $m - n$ is odd

7. (8 pts) Consider the recursively defined sequence $s_n = 4s_{n-1} - 4s_{n-2}$ where $s_0 = 1$ and $s_1 = 8$. Give an explicit formula for s_n.

8. (12 pts) Let $H = \{x \in \mathbb{N} : x \leq 100\}$, and define the function $r : H \to H$ by $r(x) = x \bmod 17$. Determine the following:

(a) $r(40)$

(b) $r(100)$

(c) $r^{\leftarrow}(6)$

(d) Is r a one-to-one function?

(e) Is r onto?

9. (8 pts) Find a recursive formula for $s_n = (-2)^n + 3 \cdot (2)^n$. *Be sure to include the initial conditions.*

10. (10 pts) Prove or disprove: If x is a rational number, and y is irrational, then $x + y$ is irrational.

11. (10 pts) Prove that $P = (n+1)(n^2)(3n-1)$ is even for all integers n.

12. (10 pts) Let $s_{2n} = 2s_n + 3, s_1 = 1$.

(a) Recursively find s_4 and s_8.

(b) Give an explicit formula for s_{2^m}. Check that your formula is consistent with part (a).

13. (6 pts) Give an explicit formula for a_{2^m} when $a_{2n} = 2a_n + 5n + 3$ and $a_1 = 0$.