**1.** (15 pts) Prove or disprove the following.

(a) $2n^5 + n^3 + 8n = O(n^5)$

(b) $n^2 - 18n = O(n)$

(c) $2^{\log_4 n} = O(n)$

**2.** (10 pts) Prove or disprove the following.

(a) $n^2 + \sqrt{n} = \Omega(n)$

(b) $2n^3 - n^2 + 24 = \Theta(n^3)$

**3.** (8 pts) Calculate (and simplify) the exact sum of the following.

(a) $\displaystyle\sum_{i=0}^{6} 2^i$

(b) $\displaystyle\sum_{k=1}^{9} \log_{10}\left(\tfrac{k+1}{k}\right)$

**4.** (8 pts) Calculate (and simplify) the following.

(a) $\displaystyle\prod_{n=2}^{5} (n^2 - 1)$

(b) $\displaystyle\prod_{k=1}^{n} \frac{k}{k+1}$ (your answer will have $n$ in it)

**5.** (8 pts) With an input of size $n$, algorithm A requires $1000\sqrt{n}$ operations and algorithm B requires $5n$ operations.

(a) Which algorithm is more efficient in the long run?

(b) For which values of $n$ is algorithm A at least as efficient as algorithm B?

(c) For which values of $n$ is algorithm B at least twice as efficient as algorithm A?

**6.** (5 $\frac{\text{pts}}{}$) Consider the recursively defined sequence given by

$$a_0 = a_1 = 2, \text{ and } a_n = a_{n-1} + (a_{n-2})^2 \quad \text{for } n \geq 2.$$

Determine $a_5$.

**7.** (10 $\frac{\text{pts}}{}$) Prove (by induction) that $n^2 > n + 1$ for integers $n \geq 2$

**8.** (10 $\frac{\text{pts}}{}$) Prove that $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ for all integers $n \geq 1$

**9.** (10 $\frac{\text{pts}}{}$) Prove that for any positive integer $n \geq 2$, $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\cdots\left(1 - \frac{1}{n^2}\right) = \frac{n+1}{2n}$