1. (20 pts) Consider the function $h : \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by $h(n) = \big|\{k \in \mathbb{N} : k|n\}\big|$. That is, $h(n)$ gives the number of divisors of n. Determine and explain the following.

(a) Make a table of values for $h(n)$ for $2 \leq n \leq 12$. (No explanation needed)

(b) Determine $h(2^m)$.

(c) Determine whether h is a one-to-one function.

(d) Determine whether h is an onto function.

(e) Describe the set $h^{\leftarrow}(2)$.

2. (5 pts) Let $g(x) = 1 - \lceil x \rceil + \lfloor x \rfloor$ for $x \in \mathbb{Q}$. g is the characteristic function for some subset of $\mathbb{Q}$. What is that subset?

3. (16 pts) Consider the function $d : \mathbb{Z}^+ \to \mathbb{N}$ defined by $d(n) = \lfloor \frac{14}{n} \rfloor$.

(a) Is d an injective function?

(b) What is the image of d?

(c) What is $d^{\leftarrow}(2)$?

(d) What is $d^{\leftarrow}(0)$?

4. (12 pts) Let the function $f : \mathbb{R}^+ \to \mathbb{R}$ be given $f(x) = \frac{1}{2}\ln(4x) - 5$.

(a) Is f one-to-one?

(b) Is f onto?

(c) Is f a one-to-one correspondence? If it is, determine the inverse function. If it is not, explain why not (i.e. what part of being a one-to-one correspondence fails?).

5. (10 pts) Consider the function $q : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{Q}$ given by $q(x, y) = x/y$

(a) Is q onto?

(b) Is q one-to-one?

6. ($15\,\underline{\text{pts}}$) Consider the sets $A = \{a, b, c, d\}$, $B = \{x, y, z\}$, and $C = \{1, 2, 3, 4\}$. Give an example which satisfies each set of properties, or explain why one cannot exist.
(You can give examples either with an arrow diagram or by listing ordered pairs.)

(a) A relation from A to B which is not a function

(b) An injective function from B to C

(c) A surjective function from B to A

(d) A one-to-one correspondence from C to A

(e) A function from C to B which is onto

7. ($8\,\underline{\text{pts}}$) Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sin(x) + \cos(x)$.

(a) Is f a one-to-one function?

(b) What is the image of f? (This takes a little work)