1. (12 $\frac{\text{pts}}{}$) List 4 elements of each of the following sets. Let $\mathbb{R}$ be the universal set.

(a) $\{x \in \mathbb{N} : x^2 \geq 5\}$
(b) $\{n \in \mathbb{Q} : \log_2(n) \in \mathbb{Z}\}$
(c) $\overline{\mathbb{Q}}$
(d) $\{n \in \mathbb{N} : n^2 + 1 \text{ is prime}\}$
(e) $\mathbb{Q} - \mathbb{Z}$
(f) $\{2^n - 1 : n \in \mathbb{Z}^+\}$

2. (21 $\frac{\text{pts}}{}$) Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

(a) $A \cup B$
(b) $A \cap C$
(c) $A - B$
(d) $B - \mathbb{Z}^+$
(e) $A \oplus B$
(f) $C \cap [0, 6)$
(g) $B \oplus C$

3. (6 $\frac{\text{pts}}{}$) Consider the sets $A = \{a\}$, $B = \{b, c, d, e\}$, and $C = \{A, B\}$. Determine

(a) $\mathcal{P}(\mathcal{P}(A))$
(b) $\mathcal{P}(C)$

4. (9 $\frac{\text{pts}}{}$) Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

(a) $A \cap B \cap C$
(b) $A \cup B \cup C$
(c) $A \oplus B \oplus C$

5. (12 $\frac{\text{pts}}{}$) Consider the sets $R = \{a, b, c, d, e\}$, $S = \{a, b, y, z\}$, and $T = \{x, y, z\}$. Let $U = \{a, b, c, d, e, \ldots, y, z\}$ be the universal set for R, S, and T. Determine the cardinality (the numbers of elements) of each of the following.

(a) $R \cup T$
(b) $R \times S$
(c) $S \times T \times \{0, 1\}$
(d) $\overline{S}$
(e) $\mathcal{P}(R)$
(f) $\mathcal{P}(\mathcal{P}(S))$

6. (9 pts) Let $S = \{0, 1, 2, 3\}$ and $T = \{1, 3, 5\}$.

(a) List the elements of $S \times T$.

(b) List the elements of the set $\{(m, n) \in S \times S : m \leq n\}$.

(c) How many elements are in the set $\{(a, b) \in \mathbb{N} \times T : a < b\}$?

7. (8 pts) Define the following sets: $A_i = \{n \in \mathbb{Z}^+ : i \mid n\}$
$B_i = \{n \in \mathbb{Z}^+ : n \mid i\}$

For each set below, list 4 elements of the set (if there at least 4), or all elements of the set (if the cardinality of the set is less than 4).

(a) A_2

(b) B_4

(c) A_{12}

(d) B_{72}

8. (15 pts) Define the following sets: $A_i = \{n \in \mathbb{Z}^+ : i \mid n\}$
$B_i = \{n \in \mathbb{Z}^+ : n \mid i\}$
$C_i = \{x \in \mathbb{R} : i - 1 \leq x \leq i + 1\}$
P is the set of all prime integers

Determine the following:

(a) $\displaystyle\bigcup_{i=1}^{6} A_i$

(b) $\displaystyle\bigcap_{i=2}^{10} B_i$

(c) $\displaystyle\bigcup_{i \in P} B_i$

(d) $\displaystyle\bigcup_{i=1}^{\infty} C_i$

(e) $\displaystyle\bigcap_{i=1}^{\infty} C_i$

9. (9 pts) How many elements are in each of the following sets?

(a) $S_1 = \{(a, b) \in \mathbb{N}^2 : a + b = 8\}$

(b) $S_2 = \{(a, b) \in \mathbb{Z}^2 : a + b = 8\}$

(c) $S_3 = \{(a, b) \in \mathbb{Z}^2 : ab = 8\}$