1. (10 $\frac{\text{pts}}{}$) Prove that for an integer n, $(n+3)(n-2)+5n$ is even if and only if n is even.

2. (10 $\frac{\text{pts}}{}$) Prove that $(3,5,7)$ is the only "prime triple" (set of three consecutive odd integers that are all prime).

3. (10 $\frac{\text{pts}}{}$) Prove that for any integer n, $n^3 - n(n+3) + 4$ has the same parity as n.

4. (6 $\frac{\text{pts}}{}$) Determine the value of n in each of the following scenarios:

(a) n div $13 = 4$ and n mod $13 = 5$

(b) $\gcd(n, 126) = 18$ and $\text{lcm}(n, 126) = 1386$

5. (10 $\frac{\text{pts}}{}$) Use the Prime Number Theorem to approximate the following.

(a) The number of primes less than 1000

(b) The number of 5 digit primes

(c) The percentage of integers between 72,000 and 213,000 which are prime. Give your answer to one decimal place

6. (12 $\frac{\text{pts}}{}$) Determine the value of the following:

(a) $\gcd(46, 60)$

(b) $\text{lcm}(46, 60)$

(c) $\gcd(2^3 \cdot 3^2 \cdot 7^2 \cdot 17^3 \cdot 23, 2^4 \cdot 3 \cdot 5^4 \cdot 17^2)$

(d) $\text{lcm}(3^2 \cdot 7^2 \cdot 11 \cdot 17, 2^2 \cdot 3 \cdot 5^4 \cdot 17^2)$

(e) $\gcd(55770, 55782)$

(f) $\gcd(81435, 81448)$

7. (8 $\frac{\text{pts}}{}$) Prove that if m and n are integers and $m \equiv n \pmod{p}$, then $m^2 \equiv n^2 \pmod{p}$.

8. (8 pts) Give the multiplication table for the following rings:

(a) $\mathbb{Z}_7$

(b) $\mathbb{Z}_8$

9. (6 pts) Determine the following:

(a) $12 - 35$ in $\mathbb{Z}_{41}$

(b) 43^{63} mod 11

(c) $2 \div 5$ in $\mathbb{Z}_{13}$

10. (12 pts) Determine whether the following statements are True or False, and justify (give either an explanation if true, or counterexample if false).

(a) If p and q are distinct primes, then $\gcd(p, q) = 1$

(b) If p is a prime, then $\gcd(p, n) = 1$ for all $n > p$

(c) If p is a prime, then $\gcd(p, n) = 1$ for all $n < p$

(d) If m and n are relatively prime, then at least one of m or n must be prime.

11. (8 pts) Let p be a prime number. Determine the following, and justify (with sentence(s)).

(a) How many positive integers less than p are relatively prime to p?

(b) How many positive integers less than p^2 are relatively prime to p^2?